HOW MATHEMATICS HELPS THE TECHNICIAN

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HOW MATHEMATICS HELPS THE TECHNICIAN

IN this book on algebra, you will find an extension of much of the basic work you have already learned in simple arithmetic. What we said earlier about arithmetic applies even more strongly to algebra. In other words, you can complete your courses in radio and television without needing to know anything at all about the use of algebra. On the other hand, if you do develop an understanding of the subject—no matter how slight—you will find more and more uses for it, in everyday living, as well as in radio and television work.

If you want to become a radio and television *engineer*, you will have to acquire a good knowledge and understanding of the uses of algebra. If you want to design radio and television equipment of any but the most simple kinds, you will find algebra essential for rapid and accurate work.

It is true, of course, that you can design equipment empirically—that is by trial and error—but this is slow as well as inefficient and uneconomical. Experienced engineers plan their equipment in advance, and then find out how it works after making their calculations and building it.

Every time you use Ohm's Law, you are using a very simple form of algebra, because you first use letters to indicate the operation to be performed and the known and unknown quantities. You then substitute numbers for these letters in performing the actual operation. As we shall show you in this manual, algebra is, in many cases, much simpler than arithmetic as long as you do not let the various signs of operation and the specialized algebraical symbols confuse you.

Another way of looking at algebra is to regard it as an abstract presentation of arithmetic. It is much easier, for example, to write $A - B = C$ when:

$$A = 999{,}376$$
$$B = 880{,}757$$
$$C = 118{,}619$$

than:

$$999{,}376 - 880{,}757 = 118{,}619$$

This simplification and ease of operation becomes even more apparent when we have to divide or multiply large numbers.

In the reference text on simple arithmetic, we discussed and explained the simple and ordinary processes of addition, subtraction, multiplication, and division. Then we talked about square roots and squaring numbers. If you are not sure exactly what is meant by a square root, or squaring a number, go back and read these sections again, for you will find that these terms are used very frequently in algebra.

In many cases in algebraical operations, decimals are not used; however, in final calculations involving the determination of an absolute answer, after an algebraical expression has been reduced to its simplest form, decimals may have to be used. You should be sure that you at least understand them before tackling problems of this type.

We explained percentages, and showed you how to convert from a whole unit to thousandths and millionths in your first reference text on mathematics. These, along with significant figures and practical meter readings are all part of the foundation necessary for a good understanding of mathematics in general.

In this manual we are going to tell you how to add and subtract algebraically, and how to perform multiplication and division by using logarithms. You will also learn to use a slide rule.

When you have finished studying this manual, you will have a good basic knowledge and understanding of algebraical fundamentals and the way in which radio and television technicians handle the many, complicated formulas they use.

Positive and Negative Numbers

When you started arithmetic you dealt only with *positive* numbers. That is, numbers such as 1, 2, 3, 4, etc. You saw the method of handling positive numbers. In algebra, we handle positive numbers, or letters representing positive numbers, in exactly the same way as we handle the numbers in arithmetic. For example, in arithmetic you learned that if you add 27 and 33, the answer is 60.

You will note that in this example we performed normal addition of positive numbers, and our answer was a positive number. We knew they were positive numbers because there was no negative sign in front of them. This may sound like a very obvious statement, but it is a basic rule in algebra and one that you *must* remember before you proceed any further. This rule applies throughout. *Any expression* (figures or letters) *that does not have a minus sign in front of it is always taken to be a positive expression.*

By making the assumption of positive value, we avoid the necessity for writing a plus sign in front of positive numbers. There is also another advantage to this system. The absence of a positive sign or any other sign in front of the number indicates that it is to be handled normally; that is, as a positive number. However, if the number is to be handled in any *special* manner the sign denoting the special handling is placed in front of it.

As we proceed into more advanced calculations, we find that we need *negative* numbers. You are already familiar, to a certain extent, with negative numbers in your work with radio. An example of the use of negative numbers is found in reference to the "C" bias on radio tubes. The "C" supply is negative with respect to the chassis, as shown in Fig. 1. We have to use negative numbers to indicate the polarity of this voltage.

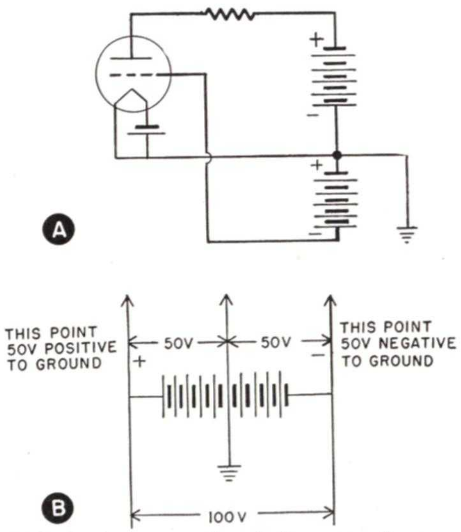


FIG. 1. An example of the use of negative numbers is found in the "C" bias in a radio circuit.

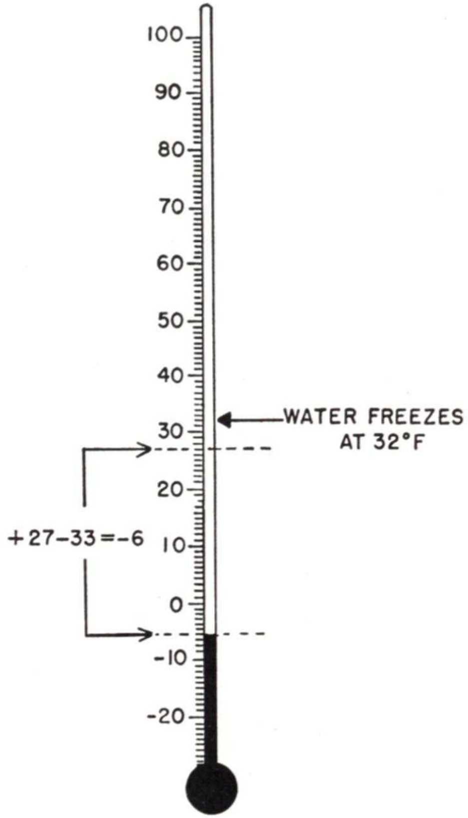


FIG. 2. The thermometer is an everyday example of the use of negative numbers.

Another example is the thermometer with which we are all familiar. Because we have temperatures lower than zero degrees Fahrenheit, we need a means of indicating such temperatures.

A thermometer is shown in Fig. 2. From 0, we go in a positive direction toward the freezing point of water (32°F) and on up to the boiling point (212°F). We can also go in a negative direction from zero.

We all know that in the summer the temperature may be over 100 degrees above zero, and in the winter up in the northern part of the country, it is not uncommon for the temperature to be ten or twenty degrees below zero. This would be indicated by a minus sign placed before the number. For example, —10, —20. These are everyday examples of negative numbers.

Negative numbers can extend just as far as positive numbers; that is, they are unlimited, so that we can have minus 1,000,000 just as easily as plus 1,000,000.

ADDITION

In handling negative numbers, there are certain special rules that we must apply. These rules are given in the following paragraphs.

Let us consider the example we gave in which we added 27 and 33. We wrote down 27 + 33 = 60.

Suppose we wanted to add +27 and —33. Adding a minus number is the same as subtracting a positive number. To see how we can subtract a larger number from a smaller number, let us go back to our thermometer. Suppose the temperature is 27° above zero, and it drops 33°. The temperature would then be 6° *below* zero. So we see that +27 and —33 are —6. This illustrates the following rule:

When positive and negative numbers are added, the smaller number is subtracted from the larger, and the answer has the same sign as the larger number.

In the example just given, if we had added —27 and +33 our answer would have been +6 instead of —6.

SUBTRACTION

Now let us consider the *subtraction* of quantities with positive and *negative* values. Again let us take the figures we used above, but this time let us assume that we want to subtract —27 from 33. We set it down like this:

33 — (—27)

You will notice that we have placed parentheses around the 27 and put a minus sign inside the parentheses immediately in front of the 27 to show that 27 is a minus quantity, and another minus sign outside the parentheses to show that the minus quantity is being subtracted.

Another way of writing this expression arithmetically is as follows:

$$
\begin{array}{r}
33 \\
- \quad -27 \\
\hline
60
\end{array}
$$

Here we write it as a simple arithmetic problem with a minus sign in front of the 27 to show that it is a minus quantity and *another minus sign* in front of the whole problem to show subtraction is to be performed.

Now here is a most important rule for you to remember when subtracting algebraical quantities—numbers having negative values, or, of course, expressions consisting of combinations of letters and figures or of letters alone:

To subtract algebraical quantities, change the sign of the lower line and add.

In other words in the example just given the "lower line" (—27) has a negative sign. Therefore, we change it to a plus sign, following the rule, and add. Thus we say +27 and +33 (we know it is +33 because there is no negative sign in front of it) = 60.

Here is another example: Subtract —27 from —33. Set it down:

$$
\begin{array}{r}
-33 \\
- \quad -27 \\
\hline
-6
\end{array}
$$

We change the sign of the lower line, so we have +27, and add. This means that we add +27 to —33. Here we must go back to our rule for adding positive and negative numbers, which says we subtract the smaller number from the larger, and the answer has the same sign as the larger number.

On the other hand, if we had subtracted —33 from —27 our answer would have been +6:

$$
\begin{array}{r}
-27 \\
- \quad -33 \\
\hline
6
\end{array}
$$

When we changed the sign of the lower line (—33) it became +33, and adding +33 and —27 gave us +6.

Here are a few examples for you to try out. The answers are on page 29, so that you can check your work.

$$200 - (-83) = ?$$
$$937 + (-133) = ?$$
$$-999 - (-1163) = ?$$

PARENTHESES

This discussion of subtraction and addition of positive and negative quantities illustrates the need for, and use of, parentheses in algebraical operations. There are two rules which apply to the use of parentheses:

If you encounter a series of terms enclosed in parentheses ( ) it means that these terms are to be considered as being one quantity. The plus or minus signs enclosed by the parentheses are governed by the positive or negative sign immediately *preceding* the parenthesis to the left.

Getting rid of the parentheses is called "reducing" the expression. When we reduce an expression, we must be sure to watch the positive and negative signs. There are certain rules to remember.

If the expression enclosed in parentheses is preceded by a minus sign, we must change the sign of every term within the parentheses in order to get rid of the parentheses.

For example, suppose we had the expression 16 — (8 + 6). To remove the parentheses, we change the signs within, and we have 16 — 8 — 6. This equals 2. We could also have performed the operation within the parentheses first, and then removed them. In that case we would have had 16 — (14); 16 — 14 = 2. As you can see, we get the same answer either way.

When an expression in parentheses is preceded by a plus sign, we do not change the signs contained in the parentheses when we remove the parentheses.

For example:

$$3 + (8 + 6) = 17$$

Because the parenthesized expression is preceded by a plus sign, when we remove the parentheses, we do not change the plus sign in front of the 6. Thus we can re-write the expression:

$$3 + 8 + 6 = 17$$

MULTIPLICATION

Multiplication and division using letters instead of numbers are no more difficult than multiplication and division of ordinary numbers, and the usual rules of arithmetic are applied. As a matter of fact, a lot of people find calculations in algebra even simpler than arithmetic because decimals are not very frequently used.

Since we are talking about multiplication and division, let us see just what happens when we multiply and divide mixed numbers; that is when we multiply and divide expressions containing numbers with mixed (positive and negative) signs.

We will talk about simple arithmetic multiplication and division first, because whatever rules apply to arithmetic also apply to algebraical calculations.

If we multiply 9 by 7, the answer is 63. However, 9 multiplied by —7 is —63.

This shows us a very important rule. It is that multiplying a plus quantity by a minus quantity always produces a minus or negative quantity as the answer. It doesn't matter whether you multiply +9 by —7 or —9 by +7, the answer will be —63. To illustrate this, suppose the temperature dropped 3 degrees per hour for 4 hours. The total temperature change would be —3 × 4 or —12°.

However, if we multiply *two negative* numbers, the answer is *positive*. —9 times —7 is +63, or, as positive numbers are usually written, 63.

We can now see three rules for multiplication:

1. If two positive numbers are multiplied, the answer is always positive.
2. If two negative numbers are multiplied, the answer is always positive.
3. If a negative number and a positive number are multiplied, the answer is always negative.

ALGEBRAIC DIVISION

The rules for dividing numbers of like and unlike signs are similar to the rules for multiplication.

When we divide a positive number by a positive number, the answer is always a positive number.

For instance, consider the expression 156 divided by 12. The answer to this problem is (+)13.

Now suppose that we had to divide —156 by (+)12. Because one of the two numbers is a minus quantity, the answer will be —13. On the other hand, if the problem had been 156 divided by —12 the answer would still have been —13.

However, if the problem had been —156 divided by —12 the answer would be +13.

You can always check the sign and value of your answer by multiplying the divisor by the quotient; e.g., —12 × +13 = —156.

Thus we see from the foregoing that the rules for algebraical division are very similar to those for algebraical multiplication. These are the rules:

1. In a problem in division, if both the dividend and the divisor are positive, the answer will be positive.

2. If either the dividend or the divisor is negative, but not both, the answer will be negative.

3. If both the dividend and the divisor are negative, the answer will be positive.

Using Letters Instead of Figures

Now that we have seen, by using easily understood figures and numerical terms, how the fundamentals of addition, subtraction, multiplication, and division, are handled algebraically, we will see how the same rules apply when we use letters instead of figures.

Algebra is useful to the radio and television technician because it is possible to use letters or symbols to represent quantities in a general manner. Such letters or symbols are known as *general*, or *literal*, numbers. We have already discussed Ohm's Law, in which we use letters to represent quantities, and this is comparatively simple to understand.

If you remember that the letters a, b, c, etc., express values or quantities, *and in reality stand for figures,* you will have no difficulty in understanding simple algebra.

We could write $5 + 3 + 9 = 17$, or we could write $a + b + c = z$, when $a = 5$, $b = 3$, $c = 9$, $z = 17$.

This is very convenient, for sometimes we do not want to write out long numerical expressions, especially if they have to be repeated many times. It is much easier to write down a, b, or c, etc. than numbers with many figures.

Here is an example of algebraic addition of terms using letters.

Add $x + 3y + 4z$; $2x + y + z$; and $5x + 2y + 2z$. Each of these three terms can be enclosed in parentheses as shown: $(x + 3y + 4z) + (2x + y + z) + (5x + 2y + 2z)$

Since the signs outside the parentheses are all plus, we can remove the parentheses without changing the signs inside them. We then have: $x + 3y + 4z + 2x + y + z + 5x + 2y + 2z$

Now that the parentheses have been removed, we can add similar terms, that is, those having the same letter. For example, all the x's, all the y's, and all the z's. Since there are no subtraction signs, we add. Therefore, taking the x's first, we find we have $x + 2x + 5x$. When handling similar terms, the easiest method is to write down the term, in this case x, and then consider the purely numerical portion of the term.

Going back, we have 5, 2, and 1 (in practice we never write 1x, but we know that when a letter stands alone without a number, it means that there is only one).

Similarly, adding up the y's, we get $3y + y + 2y = 6y$.

Adding the z's, we get $4z + z + 2z = 7z$.

So our answer comes out $8x + 6y + 7z$. This is as far as we can go in combining terms in this particular expression.

If we had the same expression, but substituted minus signs for all the plus signs inside the parentheses, we would have had the following: $(x - 3y - 4z) + (2x - y - z) + (5x - 2y - 2z)$.

Again, since the signs outside the parentheses are positive, we can remove the parentheses without changing the signs within, and we would have: $x - 3y - 4z + 2x - y - z + 5x - 2y - 2z$.

When we combined similar terms (those with the same letter), we would have:

$$x + 2x + 5x = 8x$$
$$-3y - y - 2y = -6y$$
$$-4z - z - 2z = -7z$$

so our answer would be:

$$8x - 6y - 7z$$

So far we have shown examples in algebraic addition in which the terms were spread along the line. The following example shows how algebraic expressions would be arranged in columnar form.

Find the sum of: $2m + k - 4a - 2m + 6k + a + 3c + m - 3k + 3a$.

Arranging these three expressions in columnar form to perform the addition, we set it out with similar terms in a column, as follows:

$$\begin{array}{rrrr} 2m & +\ k & -\ 4a & \\ -\ 2m & +\ 6k & +\ a & +\ 3c \\ m & -\ 3k & +\ 3a & \\ \hline m & +\ 4k & & +\ 3c \end{array}$$

You will notice in this expression that the second line contains four terms, including an extra one, 3c. There are no terms containing "c" in any of the other lines, so it stands in a column by itself.

Now we add each column separately. When we perform the addition we get the answer $m + 4k + 3c$. The a term is cancelled, so we have zero for a. $(-4a + a + 3a = 0)$. If one term in an algebraical operation is completely cancelled, as a was in this example, we do not put zero, but leave it out.

MULTIPLICATION OF LETTERS

We are now coming to multiplication in algebra. This is no more difficult than addition and subtraction, and the ordinary rules of arithmetic apply, no matter whether you are handling numbers or letters. However, it is important that you *do* understand how we handle letters when multiplying algebraically.

Let us assume that we want to multiply 5a by 3a. We set the problem down as we would a straight arithmetic problem, that is:

$$5a \times 3a = 15a^2$$

The numerical parts of the expressions are treated merely as numbers, so we multiply 5 by 3, which equals 15, and place this as the numerical part of the answer.

Now we look at the letters; $a \times a$ is not 2a, but a^2. This then is the letter part of the answer. So our answer is $15a^2$.

This leads us now to a very important point to remember when performing algebraical multiplication: when multiplying algebraical terms together, add the indexes (the index is the little number in the upper right hand side of a term showing whether it is squared, cubed, etc.).

Thus if we have to multiply $2a^4 \times 5a^3$ the answer would be $10a^7$. We get this by multiplying the 2 and 5 together, which is 10. Then to multiply $a^4 \times a^3$ we add the indexes, 4 and 3, and we obtain 7. Place this in the proper index position, and the expression becomes $10a^7$—described as "10a to the seventh."

If there is no index written down, it is assumed to be 1. Thus $a \times a$ is the same as $a^1 \times a^1$ which is a^2.

When we multiply two letters, we simply write the letters next to each other. For example, ab equals $a \times b$. This can also be written with a dot between the a and the b—not on the line like a decimal point, but above the line: $a \cdot b$.

$$3a \cdot 4b = 12ab$$

As we have learned, if we multiply a positive number by a negative number, the answer is always negative. This also is true when letters are used. The product of two terms with similar signs is positive, and the product of two terms with unlike signs is negative.

$3a \times -a$ equals $-3a^2$

Sometimes we have to multiply compound numerical expressions.

For example: $a + 3$ multiplied by $a + 6$. To do this we set the terms down one under the other as for ordinary multiplication, and proceed to multiply by each term of the multiplier in turn.

$$\begin{array}{rrr} a & +3 & \\ a & +6 & \\ \hline a^2 & +3a & \\ & +6a & +18 \\ \hline a^2 & +9a & +18 \end{array}$$

Now we multiply each term of the upper number by each term of the lower number, starting at the left. First we multiply $a \times a$, this gives a^2; we write this below the line; next we multiply $a \times 3$. This gives us $3a$. Since both numbers are positive, we write $+3a$. Now we multiply each term in the upper number by each other term in the lower number, and we have $6 \times a = 6a$. We write this in a column under the similar term in the first product ($3a$). Now we multiply 6×3. The answer is 18. This is not similar to any term in the first product, so we write it down in another column. Now we add, as for ordinary multiplication.

Let us take another example; suppose we had to multiply $a + 3$ by $a - 6$. This would be worked out as shown below.

$$\begin{array}{rrr} a & +3 & \\ a & -6 & \\ \hline a^2 & +3a & \\ & -6a & -18 \\ \hline a^2 & -3a & -18 \end{array}$$

If we had: $(a + 3) \times (-a - 6)$, the answer would be:

$$\begin{array}{rrr} a & +3 & \\ -a & -6 & \\ \hline -a^2 & -3a & \\ & -6a & -18 \\ \hline -a^2 & -9a & -18 \end{array}$$

Now let us take a simple expression that does not include numerals. Let us multiply $a + b$ by $a + b$.

You will see that the answer comes to:

$$\begin{array}{rrr} a & +b & \\ a & +b & \\ \hline a^2 & +ab & \\ & +ab & +b^2 \\ \hline a^2 & +2ab & +b^2 \end{array}$$

Here is a little test for you. What would the answer be if we multiply $a + b$ by $a - b$? See page 29 for the correct answer.

SUBTRACTING LETTERS

You have already learned how to subtract positive and negative numbers. The rules are the same for subtracting letters or symbols.

Suppose we had the problem:
$(8a + 6b - 5c) - (6a - 3b + 2c)$

The first thing to do is to remove the parentheses. Remember that when an expression in parentheses is preceded by a plus sign, the signs within are not changed when the parentheses are removed, but when the expression is preceded by a *minus* sign, the signs within the parentheses *are* changed when the parentheses are removed.

In our problem, there is no sign preceding the first expression, so we assume it to be $+$. Thus we have:

$8a + 6b - 5c - 6a + 3b - 2c$

Notice that we changed the signs before all the terms in the second set of parentheses because the parentheses were preceded by a minus sign.

We can now collect similar terms, and perform the operation indicated. Thus, we have $8a - 6a + 6b + 3b - 5c - 2c$. When arranging terms like this, it is a good idea to check each expression with a pencil as it is moved from one line to the next. Be sure that one term is not overlooked or used twice! The answer is $2a + 9b - 7c$.

There is another way of performing this operation and it is usually preferred, since it is more convenient and one is less likely to make a mistake. The expressions are arranged in columnar form, just as they were for addition. Thus we would write down the following *without making any change except dropping the brackets.*

$$\begin{array}{lrrr} & 8a & +6b & -5c \\ \text{Minus} & 6a & -3b & +2c \\ \hline & 2a & +9b & -7c \end{array}$$

In this example you will see we wrote down the number to be subtracted just as it appeared in the beginning of the problem. The rule for subtracting in algebra is to change the sign of the lower line, and add. Thus mentally, $6a$ became $-6a$; adding $-6a$ and $+8a$ leaves $+ 2a$.

The term $-3b$ becomes $+ 3b$; this added to $6b$ gives us $9b$.

Finally, the last term, $+2c$ now becomes $-2c$, and adding $-2c$ to $-5c$ gives a total of $-7c$. We do not physically change the signs, but the operation is performed mentally.

Here are three examples for you to solve.

(1) Subtract $4a - 8b + c$ from $-8a + 3b + 6c$.

(2) Subtract $-ab + 3d - 4y$ from $6ab - 7d + 10y$.

(3) Subtract $8x^2y - 15xy^2 - 10\,xyz$ from $-4x^2y + 6xy^2 - 5xyz + 3a$.

Remember when you do the last problem that terms are similar only when the letters are *exactly* the same. For example $8x^2y$ and $-4x^2y$ are similar terms, but $8x^2y$ and $-15xy^2$ are not. So this problem would be set up like this:

$$\begin{array}{rrrr} -4x^2y & +6xy^2 & -5xyz & +3a \\ 8x^2y & -15xy^2 & -10xyz & \\ \hline -12x^2y & +21xy^2 & +5xyz & +3a \end{array}$$

DIVIDING LETTERS

Division in algebra is probably a little more complicated than multiplication, just as in arithmetic, but even so, by following the rules for division you will have no difficulty in understanding it. Let's look at the following example: $a^2b^2 + 3ab + 2 \div ab + 1$.

Our first step is to divide the first term of the dividend by the first term of the divisor; thus, ab goes into a^2b^2 "ab times." So we put ab as the first term in the answer above the line.

$$\begin{array}{r|l} & ab \\ \hline ab + 1 & a^2b^2 + 3ab + 2 \end{array}$$

Now we multiply the *whole divisor* by ab and write the product $a^2b^2 + ab$ under the dividend.

$$\begin{array}{r|l} & ab \\ \hline ab + 1 & a^2b^2 + 3ab + 2 \\ & a^2b^2 + ab \end{array}$$

Subtract this from the dividend and bring down the next term, (just as in simple arithmetic):

$$\begin{array}{r|l} & ab \\ \hline ab + 1 & a^2b^2 + 3ab + 2 \\ & a^2b^2 + ab \\ \hline & \quad 2ab + 2 \end{array}$$

Now divide 2ab by ab (the first term of the divisor) and multiply the whole of the divisor $(ab + 1)$ by the result (2). Thus we have the expression:

$$\begin{array}{r|l} & ab + 2 \\ \hline ab + 1 & a^2b^2 + 3ab + 2 \\ & a^2b^2 + ab \\ \hline & \quad 2ab + 2 \\ & \quad 2ab + 2 \\ \hline \end{array}$$

We can check the correctness of our answer by multiplying the quotient by the divisor. Thus we have:

$$\begin{array}{r} ab + 2 \\ \times \quad \underline{ab + 1} \end{array}$$

Here is the problem set up for you to practice multiplication, and complete the proof. Your answer should be the same as the dividend in the original problem.

Here is another example in which the dividend has one more term than the divisor:

$$\begin{array}{r|l} & \quad a + 1 \\ \hline 2a^2 - 5a - 1 & 2a^3 - 3a^2 - 6a - 1 \\ & \underline{2a^3 - 5a^2 - \quad a} \\ & \quad + 2a^2 - 5a - 1 \\ & \quad \underline{2a^2 - 5a - 1} \end{array}$$

In this case we first divide $2a^2$ into $2a^3$. This goes "a times," so we write "a" in the quotient. We now multiply the rest of the expression by "a" and set down the results under like terms in the dividend. Thus "a" multiplied by —1 is —a.

Subtracting this line from the quotient leaves $+ 2a^2 - 5a - 1$.

The first term of the divisor ($2a^2$) goes into the first term of this expression once, so we multiply the whole of the divisor by 1 and write the product under the new dividend, at the same time writing +1 in our answer. +1 times $2a^2$ equals $2a^2$, and 1 times —5a equals —5a, and +1 times —1 is —1. We write these in the proper columns and subtract. The remainder is zero.

Let's check our division by multiplying the quotient by the divisor. It should equal the dividend.

$$\begin{array}{r} 2a^2 - 5a - 1 \\ \underline{a + 1} \\ 2a^3 - 5a^2 - a \\ 2a^2 - 5a - 1 \\ \hline 2a^3 - 3a^2 - 6a - 1 \end{array}$$

EXPONENTS AND ROOTS

Although you probably have not realized it, in the preceding paragraphs you were actually handling exponents or indexes. If we write "a multiplied by a," we can call the answer, "a^2," or "a to the second power." Similarly a multiplied by a, multiplied by a, may be expressed as "a^3" or "a to the third power."

If we have a higher power than 3; for example a^8 we would call it "a to the eighth power."

We say that the base, in this case a, is raised by the exponent to the value indicated. In the example just given, a^8 means $a \times a \times a \times a \times a \times a \times a$.

If no exponent is indicated, then it is understood that the exponent is 1, so writing a is the same as writing a^1.

If we have to multiply a^2 by a^4 by a^{10} all we do is add the exponents. Thus the answer to this problem is a^{16}.

If we have to perform division of one term by another with the same letter or letters, we simply subtract the smaller exponent from the larger, for example:

$$(a^3 \times a^3) \div (a^2 \times a^3) = a^6 \div a^5 = a^1 \text{ or } a$$

Another example:

$$(a^6 \times a^3) \div (a^2 \times a^4) = a^3$$

Simplifying the problem:

$$\frac{a^9}{a^6} = a^3$$

Subtracting the exponent 6 from the exponent 9 leaves 3, so the exponent in the answer is 3. Thus the answer is a^3.

Suppose the expression had been

$$(a^2 \times a^3) \div (a^4 \times a^3) = \frac{a^5}{a^7}$$

In this case we cannot subtract the divisor from the dividend. So we subtract the smaller exponent from the larger, and place a negative sign in front of the answer. In this case we subtracted the exponent 5 from 7; thus the answer is 2. Because we subtracted in reverse, we placed a minus sign in front: —2.

The answer is: $\frac{a^5}{a^7} = a^{-2}$

We can check this by multiplying the divisor by the quotient:

$$a^7 \times a^{-2} = a^5$$

This shows us another way of multiplying by use of exponents. When multiplying expressions involving terms with negative and positive exponents, you merely add the exponent, following the instructions of the terms:

$$a^2 \times a^{-10} \times a^6 \times a^8 \times a^{-4} = a^{2-10+6+8-4} = a^2$$

Perhaps you have noticed a very interesting point from the above. It makes algebraical arithmetic very much simpler at times. Multiplying by a negative exponent is the *same* as dividing by a positive exponent of the same value.

$$+a^3 \times a^{-9} = a^{-6}; \text{ and } \frac{+a^3}{+a^9} = a^{-6}$$

(subtracting smaller exponents from larger and changing sign).

It is important to realize that a negative number cannot have a square root, since in order to obtain a negative sign in front of a number, we have to multiply negative and positive terms together. For example,

$$+4 \times +4 = +16$$
$$-4 \times -4 = +16$$

(Like signs give a positive answer.)

$$-4 \times +4 = -16$$

(Unlike signs give a negative answer.)

In the first expression, $+4 \times +4$, the two terms are completely identical in that they both have the same sign, and the same numerical value.

In the second case also, each term has the same sign (—) and the same numerical value, therefore they are identical. Their product also has a positive value.

But in the third line we have two numbers which have the same figure (4) but in actual fact they are very different, since one is a — quantity and the other is a + quantity. Since a square root has to be composed of two absolutely identical quantities, you cannot say that a positive and a negative quantity—even though they have the same *apparent* numerical rating—constitute a square root.

In general, every number has two square roots equal in magnitude but opposite in sign, one is positive and one is negative. The positive square root is known as the "principal," and if no sign precedes the square root we always assume it to be positive. Thus the square root of a^8 can be $\pm a^4$ (we call it "plus or minus a to the fourth"). We know that this is true because

$$a^4 \times a^4 = a^8$$

or

$$-a^4 \times -a^4 = a^8$$

If no sign precedes the radical sign we always assume that the positive root is required:

$$\sqrt{16} = +4$$

but

$$-\sqrt{16} = -4$$

Generally speaking, you will not be concerned in your radio, television, and communications work, with mathematics. Nevertheless, there may be occasions when a nodding acquaintanceship with some of the rules of mathematics can prove very valuable. Quite often, when you are talking to a prospective employer about a job, little, apparently unrelated, bits of knowledge which you have acquired in your courses, such as this, are sufficiently impressive to make him select you rather than another candidate whose surface qualifications are similar.

Calculation of Resistance Values

The basic electrical characteristics which determine the choice of a resistor for a particular application are:

(a) Its Resistance (R) in OHMS.
(b) The Current (I) in AMPERES flowing through it.
(c) The Voltage (E) in VOLTS applied across it.
(d) The Power (P) in WATTS dissipated by it.

These values are interrelated, and when any two of them are known, the others can be found. Fig. 3 shows a chart that can be used, and the following formulas show how these values can be calculated:

(1) When E and I are known, then

$$R = \frac{E}{I}; \text{ and } P = E \times I.$$

Example: What resistance will produce a bias voltage drop of 20 volts when a current of 50 milliamperes (.050 ampere) flows through it, and what is the power dissipation?

$$\text{Answer: } R = \frac{20}{.050} = 400 \text{ ohms}$$
$$P = 20 \times .050 = 1 \text{ watt}$$

(2) When E and R are known, then

$$I = \frac{E}{R}; \text{ and } P = \frac{E^2}{R}$$

Example: What current will flow through a 400-ohm resistor when 120 volts are applied across it? How many watts will be dissipated?

$$\text{Answer: } I = \frac{120}{400} = 0.3 \text{ ampere}$$
$$P = \frac{120 \times 120}{400} = 36 \text{ watts}$$

(3) When I and R are known, then $E = IR$ and $P = I^2R$

Example: An ammeter connected in series with a 35-ohm resistor in an electrical circuit indicates a current flow of 3 amperes. What is the voltage drop across the resistor and the wattage dissipation of the resistor?

Answer: $E = 3 \times 35 = 105$ volts
$P = 3 \times 3 \times 35 = 315$ watts

(4) When P and E are known, then

$$I = \frac{P}{E}; \text{ and } R = \frac{E^2}{P}$$

Example: A power-operated device is rated 60 watts at 110 volts. What current will it draw, and what is its effective resistance?

$$\text{Answer: } I = \frac{60}{110} = 0.545 \text{ ampere}$$
$$R = \frac{110 \times 110}{60} = 201.7 \text{ ohms}$$

(5) When P and I are known, then

$$E = \frac{P}{I}; \text{ and } R = \frac{P}{I^2}$$

Example: A 50-watt resistor has a

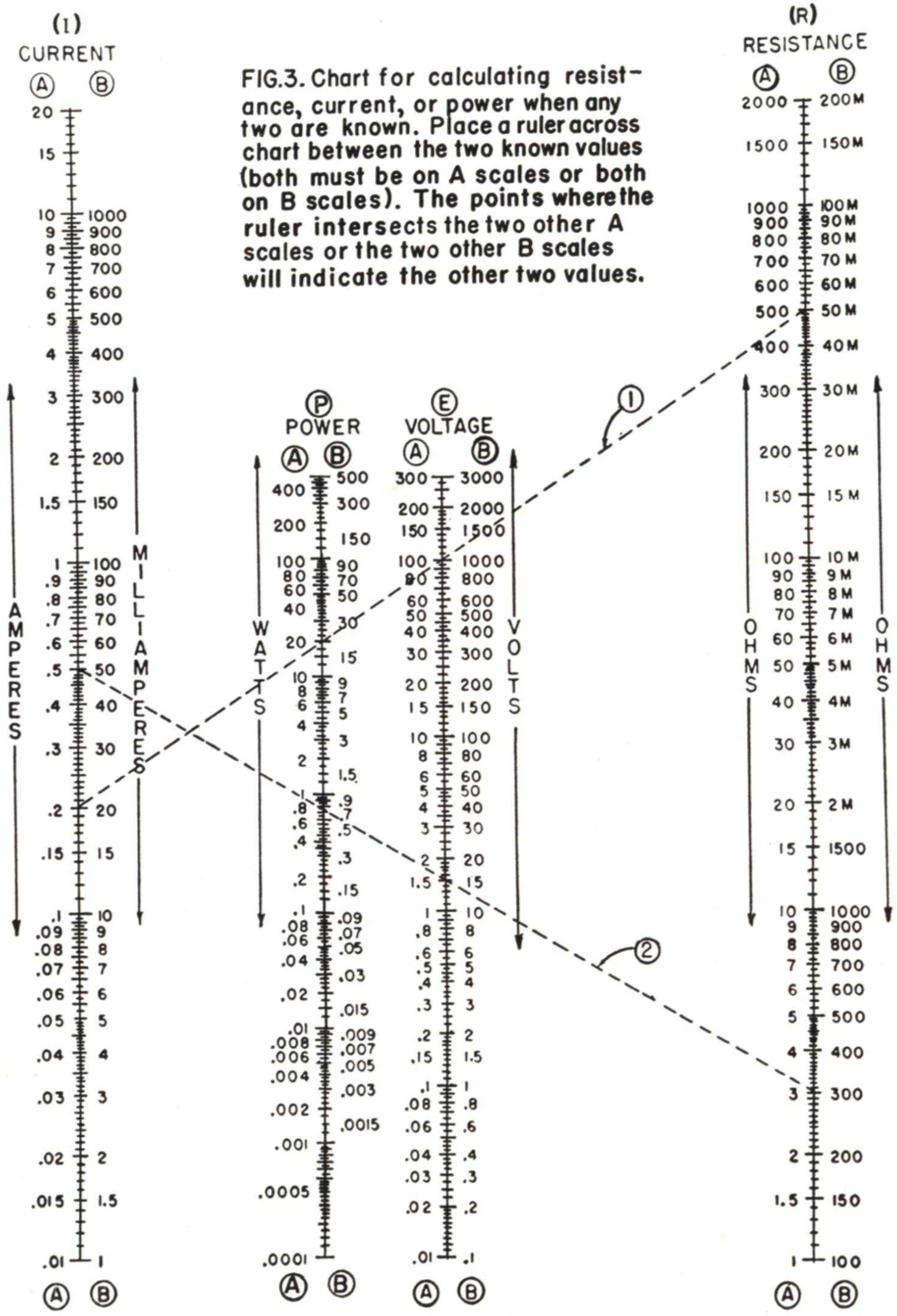


FIG.3. Chart for calculating resistance, current, or power when any two are known. Place a ruler across chart between the two known values (both must be on A scales or both on B scales). The points where the ruler intersects the two other A scales or the two other B scales will indicate the other two values.

maximum current rating of 1.58 amperes. What is the maximum voltage that may be applied across it, and what is its nominal resistance?

Answer: $E = \frac{50}{1.58} = 31.6$ volts;

$$R = \frac{50}{1.58 \times 1.58} = 20 \text{ ohms}$$

(6) When P and R are known, then

$$E = \sqrt{PR}; \text{ and } I = \sqrt{\frac{P}{R}}$$

Example: What is the maximum voltage and current that may be applied to an 8000-ohm resistor without exceeding a recommended power rating of 20 watts?

Answer: $E = \sqrt{20 \times 8000} = 400$ volts

$$I = \sqrt{\frac{20}{8000}} = .050 \text{ ampere}$$

RESISTORS IN PARALLEL

When two or more resistors are connected in parallel, the equivalent resistance of the combination is equal to the reciprocal of the sum of the reciprocals of the individual resistances.

The reciprocal of a number is 1 divided by that number. If we call the individual resistances R1, R2, R3, etc., their reciprocals will be $\frac{1}{R1}$, $\frac{1}{R2}$, $\frac{1}{R3}$, etc. Their sum will be $\frac{1}{R1} + \frac{1}{R2} + \frac{1}{R3}$ etc., and the reciprocal of their sum will be:

$$\frac{1}{\frac{1}{R1} + \frac{1}{R2} + \frac{1}{R3}}, \text{ etc.}$$

Example: What is the equivalent resistance of a combination of 30 ohms, 50 ohms, and 60 ohms connected in parallel?

$$Rt = 1 \div (\frac{1}{30} + \frac{1}{50} + \frac{1}{60})$$

In order to add $\frac{1}{30}$, $\frac{1}{50}$, and $\frac{1}{60}$, we must have a common denominator. Using 300, we have:

$$Rt = 1 \div (\frac{10}{300} + \frac{6}{300} + \frac{5}{300})$$

$$= 1 \times \frac{300}{21}$$

$$= 14.29 \text{ ohms}$$

Where the calculations involve only two parallel-connected resistors at a time, the formula may be more conveniently expressed as:

$$Rt = \frac{R1 \times R2}{R1 + R2}$$

This is just another way of expressing the same formula. This is how we arrived at it:

$$Rt = \frac{1}{\frac{1}{R1} + \frac{1}{R2}}$$

In order to add $\frac{1}{R1}$ and $\frac{1}{R2}$, we must have a common denominator, which would be R1 × R2. This would give us:

$$Rt = \frac{1}{\frac{R2}{R1 \times R2} + \frac{R1}{R1 \times R2}} \text{ or } \frac{1}{\frac{R2 + R1}{R1 \times R2}}$$

To divide 1 by a fraction, we simply invert the fraction and we have:

$$Rt = \frac{R1 \times R2}{R1 + R2}$$

If we wanted to find R2, we would write the formula:

$$R2 = \frac{R1 \times Rt}{R1 - Rt}$$

Example: What resistance would be required in parallel with a 50-ohm resistor to obtain an effective resistance of 40 ohms?

Answer: $R2 = \frac{50 \times 40}{50 - 40} = 200$ ohms.

RESISTORS IN SERIES

When the individual values of resistors connected in series are known, the total resistance of the combination is equivalent to the sum of the ohmic resistances; or:

$$Rt = (R1 + R2 + R3 + \ldots \text{ etc.})$$

You may frequently want to calculate the size of a series resistor needed in a circuit. Suppose you had an electrical appliance to be operated from a line voltage that was higher than the voltage rating of the device. You would need to increase the resistance in the circuit. Since you know that the total resistance in a circuit is equal to the sum of all the resistances in series in the circuit, you would add a resistor in series with the device to be operated.

If you knew the line voltages, EL, the voltage rating of the device, ED, and the wattage rating of the device, PD, the formula to use to find the value of resistance needed is:

$$Rs = \frac{ED(EL - ED)}{PD}$$

To see how we arrived at this formula, let's start with the formula for resistance: $R = \frac{E}{I}$. The voltage across the resistor would be the line voltage, EL, minus the voltage across the device, ED. We do not know the current, but we do know the wattage, PD, and we know the formula for current is $I = \frac{P}{E}$. Dividing PD by ED gives us the current through the device. Since the current is the same anywhere in the circuit, this is the figure we would use. So our formula would be:

$$Rs = \frac{EL - ED}{\frac{PD}{ED}}$$

To divide by a fraction, we invert it and multiply, so we have:

$$Rs = \frac{ED(EL - ED)}{PD}$$

Example: What series resistance will be required to operate a 110-volt, 50-watt device on a 130-volt power line?

Answer: $Rs = \frac{110(130 - 110)}{50}$

$= 44$ ohms.

The power dissipation (Ps) of the series resistance will be:

$$Ps = \frac{PD(EL - ED)}{ED} = \frac{50(130 - 110)}{110} = 9.09 \text{ watts.}$$

Logarithms

Mathematicians and engineers are always trying to find ways of performing operations more simply and quickly. The old methods of long division and involved multiplication frequently prove too slow and involved for the very long calculations they sometimes have to perform.

Therefore, mathematicians have developed the system of *logarithms*. As a matter of fact it is easier to do logarithms than to pronounce the name! You have already learned how to find square roots and to square numbers. In this section, you will see how by using logarithms you can do it in about a quarter of the time and space. It is effectively a short-cut method of multiplying, dividing, extracting roots, and raising numbers to desired powers.

This is the way that practical radio and television technicians calculate—they always use logarithms whenever possible as a convenient short-cut method.

THE THEORY OF LOGARITHMS

Let us begin our study of logarithms with a consideration of the simple number 10. If we multiply 10 by itself, (square it), we get 100. That is, 10×10, or $10^2 = 100$. In the same way, $10 \times 10 \times 10$, or $10^3 = 1000$, and $10 \times 10 \times 10 \times 10$, or $10^4 = 10{,}000$.

In the expressions 10^2, 10^3, and 10^4, etc., the small number to the right is the power, or exponent. And, as you know, if we were to write the number 10 with an exponent, it would be 10^1.

Conversely, if we have the number 100, the square root ($\sqrt{100}$) is 10, for $10 \times 10 = 100$. Similarly the cube root of 1000 ($\sqrt[3]{1000}$) is 10, and the fourth root of 10,000 ($\sqrt[4]{10{,}000}$) is 10, since 10^4, or $10 \times 10 \times 10 \times 10 = 10{,}000$.

Of course, all this is very simple, but it is not always quite as easy to realize that any number can be expressed in terms of 10 raised to a certain power.

Take, for example, the number 2. This could be expressed as $10^{.301}$, which says that if it were possible to multiply the number 10 by itself .301 times, the product would be 2. The exponent .301 is called the "logarithm" of the number 2. All the logarithms we shall use and discuss are based on 10; we refer to them as logarithms to the base 10. These are called "common logarithms."

Let us take another example. The number 44 can be expressed as $10^{1.6435}$. The logarithm of the number 44 is 1.6435. Notice now that the logarithm is divided into two parts—one part to the left of the decimal point, the other to the right of the decimal point. The part to the left is called the "characteristic," and the part to the right the "mantissa" of the logarithm (or log).

In the logarithm of a number, the characteristic is equal to one less than the number of figures to the left of the decimal point in the original number. Therefore, a characteristic of 1 means that there are two figures to the left of the decimal place in the number.

In other words, we know that there are two whole numbers in it, so it must lie between 10 and 99. The very important rule to remember here is that the characteristic is always one number smaller than there are whole numbers (numbers to the left of the decimal point) in the original number.

Thus if the logarithm of a number is exactly 3, the number itself is exactly 1000. Because 1000 has four places to the left of the decimal place, (or 4 whole numbers), the characteristic is 3. This is one less than the total number of whole numbers, which in turn means that the whole number must be exactly 1000.

Try to memorize the following table:

For numbers from:			Characteristic
1	to	9	0.
10	to	99	1. (10^1)
100	to	999	2. (10^2)
1,000	to	9,999	3. (10^3)
10,000	to	99,999	4. (10^4)
100,000	to	999,999	5. (10^5)

The logarithm of a decimal fraction has a negative characteristic. The presence of a minus sign in front of a logarithm would be confusing, especially in multiplying and dividing, because although the characteristic may be negative or positive, the mantissa is *always* positive. To avoid any possible confusion produced by the presence of a minus sign in front of a logarithm, mathematicians found a new place and name for the minus sign. They decided to place it *above* the characteristic of the logarithm so that it looks like this:

$$\bar{5}.937$$

You will, of course, notice that the minus sign has been removed from the front of the characteristic and placed *above* it. Instead of identifying the logarithm as "*minus* five point nine three seven" we now call it "*bar* five point nine three seven."

In other words the minus sign is placed over the top of the characteristic and called a "bar." Whenever you see a logarithm with a horizontal bar above the characteristic, you know at once that it is a negative characteristic.

In a logarithm with a negative characteristic, the characteristic is always equal to one more than the number of zeros immediately following the decimal point in the original number. Therefore, the number (called the antilog) for which $\bar{5}.937$ is the logarithm, is between .00009 and .00001. You know that since the characteristic is 5, the number contains four zeros to the right of the decimal point. Here is a table showing you the characteristics of numbers for 0.9 to .00001.

For numbers from:			Characteristic
.9	to	.1	—1. (10^{-1})
.09	to	.01	—2. (10^{-2})
.009	to	.001	—3. (10^{-3})
.0009	to	.0001	—4. (10^{-4})
.00009	to	.00001	—5. (10^{-5})

Having in mind the use and meaning of the characteristics of logs, we are now ready to work with mantissas. To obtain the mantissa of any number, we shall need a log table such as the one on pages 18 and 19. You will notice that only mantissas are given.

For an example let us start with the number 39. We know that the characteristic will be 1, because it is always equal to 1 less than the number of figures in the antilog.

Now, to find the mantissa, refer to the log tables. First find the number (39) in the N column. Since the number 39 is the complete number, follow across to the 0 column. There you will find 5911, which is the mantissa. Since you already know that the characteristic is 1, you have the complete logarithm of 39, that is, 1.5911.

If the number for which you wanted the logarithm had 3 places, you would find the first two in the N column, then follow across to the column under the third digit. For example, to find the logarithm of 399, you would find 39 in the N column, and then follow across to the 9 column, where you would find

N	0	1	2	3	4	5	6	7	8	9	P.P. 1	2	3	4	5
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	5	7	9
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	6
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4

N	0	1	2	3	4	5	6	7	8	9	P.P. 1	2	3	4	5
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	3	3
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2

6010 for the mantissa. Since the number 399 has 3 places, you know the characteristic is 2, so the complete logarithm is 2.6010.

If the number had been 3.9, our log would have been .5911. If .39, it would be —1.5911, or $\bar{1}.5911$. If .0039, the log would be —3.5911, or $\bar{3}.5911$, etc.

Here are some examples for you to try. What is the log of:

(1) 483
(2) 48.3
(3) 4.83
(4) .483
(5) .0483

The answers are on page 29.

MULTIPLICATION

In the following pages, you will see just how easily logarithms can be used in multiplication and division, and how such problems can be simplified by their use. All you have to do to multiply one number by another is add the logarithms of the numbers. Conversely, to divide, all you do is subtract the logarithm of the divisor from the logarithm of the dividend. Since logarithm is such a long word, we usually abbreviate it to log.

Suppose we want to multiply 599 by 39. We find the logarithm of 599 by first writing down the characteristic which is 2. We now take the whole number 599 and look it up in the tables.

We find 59 in the N column, and 9 in the last column of the numbers 0 through 9.

We now move horizontally to the right along the 59 line until it intersects the vertical column of figures under 9. The number is 7774. Thus the logarithm of 599 is 2.7774. We now find the logarithm of 39, which is 1.5911. We add 2.7774 to 1.5911 and get 4.3685. This is the logarithm of our answer. To find the answer, we have to find the antilog of 4.3685.

(The opposite of a logarithm is known as the antilogarithm. Thus the logarithm of 599 is 2.7774. Conversely, the antilogarithm of 2.7774 is 599.)

In our example, because the characteristic of the logarithm is 4, we know that the antilog will be between 10,000 and 99,999. Now we try to find the mantissa 3685 in the log table. The log table, you will see, starts with 0000 and goes to 9996. As you move along in each line of the log table to the right, you find that the mantissa increases. We want to find the number for which the logarithm contains the mantissa 3685. Knowing the direction in which the numbers run makes it very easy for us to see that 3685 should be in the horizontal row beside 23. Examining this row more closely, we see that 3685 would fall between the vertical column for 3 and the vertical column for 4. The number 3692 for column 4 is closer to 3685 than the number for column 3, so we use column 4.

This means that our answer is 234. This is made up of 23 from the left hand column N and the number 4 (the number of the vertical column in which the mantissa is found). Because the characteristic of the logarithm was 2, we know our number must be between 10,000 and 99,999. Therefore we add two zeros to 234 and our answer becomes 23,400.

If we had worked this out by the long method, the answer would have come to 23,361. In most cases in practical radio and television work, 23,400 would have been close enough. However, occasionally it is necessary to have four significant terms in the answer. The following paragraph shows how we can obtain an answer correct to four places.

If we wanted an answer correct to four places, we would use the last column of the log table. This is called the Proportional Parts column. To use this column we first locate the mantissa nearest to 3685, in this case the larger one, 3692. This is larger than the number we want by 7.

Now, we continue horizontally along the same line on which our mantissa is located until we find 7 in the extreme right-hand column, labeled P.P. We now go vertically up the column in which we found 7. We find that 7 is under 4; in other words the *proportional part* for 7 is 4, as read at the top of the column.

You will remember that our answer as originally found using the mantissa closest to our actual one (3692) was 23,400. To obtain a number with four significant terms we subtract the proportional part, 4, from 2340. This gives us 2336.

Now, you will remember, the characteristic was 4, which means that we have five whole numbers in the answer. If we add one zero to 2336 our answer becomes 23360—with four significant terms. (You should note that it was purely coincidental that the proportional part was 4 as well as the characteristic being 4. There is no connection between these two numbers.)

As an additional example let us solve the problem 965.43 × 83.97.

First we find the logarithm of 965.43. There are five numbers in this term and since the last one is less than 5 we disregard it. We now have four numbers with which to work. Our log table carries us through only three. Therefore we have to do something about the 4. The logarithm of 965 is 9845 from the table. We now continue along the same line until we come to the proportional parts column. In this column we locate the last figure 4, at the top. We now read down the column under the figure 4 and on the same line as 96 we find the number 2. This is added to the mantissa, making it 9847.

The log of 83.97 is 1.9241. We found it in this manner: because the final number (7) of 83.97 is larger than 5 we call the 9 a 10, and work backwards this time, from 84.00. The mantissa of 84 is 9243, but this is larger than the mantissa for the required number. We subtract our original number from the number which we used, that was closest to it (84) thus 8400 —8397 = 3. This is the figure we use to find our proportional part.

We look at the top of the column of proportional parts and find the number 3. We now drop down this column until we are level with 84. The proportional part which we read at this point is 2, so we subtract 2 from the logarithm of 84, which equals 9243 minus 2; thus our logarithm is 1.9241.

Now, to perform the multiplication, we add the logarithms, thus:

$$2.9847 + 1.9241 = 4.9088.$$

The antilog for this is 81070. We found it by locating the mantissa nearest to 9088. We find it to be 9090. The antilog for 4.9090 is 81,100 but 9088 is 2 less than the mantissa we used (9090), so we have a proportional part of 2. Therefore, we continue along the line 81 in our log table until we come to 2 in the column for proportional parts. This is the third column of proportional parts, so the number we use to find the fourth figure in our antilog is 3. Subtracting this from the number 8110 we get 8107. Since the characteristic is 4, we know that we must have five whole numbers in the answer, therefore we add one zero, and our answer becomes 81070.

If we work out the problem by arithmetic, we would obtain 81,067.1571 as our product. However, except where computations involving money are made, four significant figures are sufficient, therefore our product 81,070 is close enough for all practical purposes.

At this point you should work out a

number of multiplication problems, both by arithmetic and by logarithms. After going through the procedure a few times, and checking your work as you go along, you will begin to appreciate how easy and convenient it is to use logs.

Here are three tests for you to practice on, if you like. The answers are given on page 29, so that you can check your work if you want to.

(a) 965 × 56
(b) 23.2 × 135.9
(c) 4800.01 × 13.7

DIVISION

Division by the use of logarithms is just as simple as multiplication. As an example, let us take a problem we worked out by long division in another lesson: 969,424 ÷ 31. Disregarding the last two figures (24) in the dividend as being insignificant, the log is 5.9865.

The log of 31 is 1.4914. Subtracting these we get 4.4951, which is the log of our quotient. We find that the antilog of this is 31,270, which for practical purposes is as good as the quotient we obtained in long division, 31,271.74.

A more difficult problem would be to divide .000375 by 17. First we find the log of .000375, which is $\bar{4}.5740$, then we find the log of 17, which is 1.2304.

The next step is to subtract the logs, but the fact that the logarithm of .000375 is $\bar{4}.5740$ presents some difficulties. We cannot subtract 1.2304 from $\bar{4}.5740$ because as you learned, although the characteristic of a logarithm may be negative, the mantissa is always positive. So although the 4 is negative, the 5740 is positive, so we have to adopt a subterfuge. There are two ways of doing this. The first is rather clumsy as you will see, but it is quite commonly used.

Instead of writing $\bar{4}.5740$ we replace the $\bar{4}$ with the term 6 — 10, which is the same as —4. By writing the logarithms as shown below, we have a positive characteristic as well as a positive mantissa, and we can subtract. The extra 10 which we used to make this subterfuge possible is put at the righthand side of the line as shown below:

$$\begin{array}{rrr} & 6.5740 & -10 \\ - & 1.2304 & \\ \hline + & 5.3436 & -10 \end{array}$$

After we perform the subtraction indicated, we must subtract the ten from the characteristic, so we have —5.3436 as the logarithm of our answer. We find that the antilog is .00002206.

However there is a much easier way of performing this operation by taking advantage of the "bar." First of all we write down the problem with the bar sign over the top of the characteristic of the first term. This bar makes it plain that only the characteristic is negative.

$$\begin{array}{rr} & \bar{4}.5740 \\ \text{Minus } (+) & 1.2304 \\ \hline & \bar{5}.3436 \end{array}$$

Therefore, in the subtraction problem above, we commence by subtracting +1 from bar 4 (negative 4). Following the algebraical rules for subtraction we change the sign of the +1 to —1 and *add*, thus our characteristic becomes —5 or, as it should be called, bar 5. We now *subtract* the mantissa since that is a *positive* number and the answer comes to .3436. Thus our logarithm is $\bar{5}.3436$.

The only way in which the beginner can go wrong in using this system is if he forgets that the mantissa is always handled as a positive number even though the characteristic may be negative at times.

SQUARING NUMBERS AND FINDING ROOTS

Squaring large numbers and finding the square roots of large numbers are by no means simple tasks if ordinary methods of arithmetic are used. When powers and roots other than 2 are involved, the arithmetical procedures are extremely complicated and it is easy to make mistakes.

You know that 25^2 is another way of writing 25 × 25. It is read "25 squared." Similarly, 25^3 means that 25 is to be raised to the 3rd power, and 25^4 means that 25 is to be raised to the 4th power, that is, 25 × 25 × 25 × 25.

The radical sign $\sqrt{}$ before and over a number indicates that the square root of that number is to be found. Similarly a radical sign with a 3 or 4, (or any other number) written $\sqrt[3]{}$, or $\sqrt[4]{}$, means that we are to find the cube root or the 4th root or whatever root is indicated by the number. Remember that a root of any number is merely a number which multiplied by itself the indicated number of times produces the given number.

Multiplying 2 × 2 × 2 is comparatively easy, and of course the answer is 8. However, if we had to cube, say 819, that is, multiply 819 by 819 by 819, it would take much more time and work, and it would be very easy to make a mistake. Similarly, if we wanted to find the square root of 819, we would have to use the fairly complicated processes already described in an earlier lesson. However, using logarithms makes any problem involving raising a number to a power or reducing a number to a root extremely simple.

To square a number, all we do is multiply its logarithm by 2. To cube it, multiply the logarithm by 3, to raise a number to any given power, we merely multiply its log by that power.

We say that the "nth" power of a number is "n" times its logarithm, (n is the symbol used by mathematicians to indicate any number). To find any power of a number, merely multiply its logarithm by that power, and find the antilog of the number thus produced, in the table.

If we want to find the root of a number, we divide the logarithm of the number by the root required. Thus, if we want to find the square root of a number, we look up its logarithm in the tables and divide that log by 2. The square root is then the antilog of the number thus found.

To find the cube root, divide by 3 and to find the nth root divide the logarithm by n.

Here are 4 problems involving powers and roots. In the first we want to find the square of a number, in the second, the cube of a number, in the third, the square root of a number, and in the fourth, the cube root of a number.

(1) Find the square of 393. (393^2)
Log 393 = 2.5944
2.5944 × 2 = 5.1888
The antilog of 5.1888 = 154500

(2) Find the cube of 25. (25^3)
Log 25 = 1.3979
1.3979 × 3 = 4.1937
The antilog of 4.1937 = 15620

(3) Find the square root of 53000.
Log 53000 = 4.7243
4.7243 ÷ 2 = 2.3621
The antilog of 2.3621 = 230

(4) Find the cube root of 15620 ($\sqrt[3]{15620}$)
Log 15620 = 4.1937
4.1937 ÷ 3 = 1.3979
The antilog of 1.3979 = 25

Study these problems very carefully

and check the logs and antilogs against the log table.

Note: When you multiply by 2 or any other number to raise a number to a given power, the characteristic as well as the mantissa is multiplied; when we divide by a number to find a root we divide both the characteristic and the mantissa by the root required.

The Principle of the Slide Rule

As you have already noticed, mathematicians are always looking for ways to simplify the long involved calculations they so frequently perform. Although the use of logarithms offers a quick and easy way of performing multiplication and division, they found a way of carrying out these involved problems by means of a simple mechanical device which eliminated the need to do any writing at all—except of course, for writing down the answer. They called it—the *Slide Rule.*

Don't be frightened by the name slide rule or by any preconceived ideas you may have of visions of geniuses poring over slide rules!

You will find the slide rule used by all manner of people. The slide rule is based on the use of logarithms, and it is simply a mechanical way of adding and subtracting them. You work with ordinary numbers which are laid out in a *logarithmic* pattern on the slide rule scales.

You will remember that characteristic curves of radio and television receivers are usually charted logarithmically, that is, on graph paper with the divisions laid out according to the logarithmic laws. On a logarithmic scale, divisions become smaller as you go from one to ten. Between 1 and 2, we find the largest division, and the distance between consecutive numbers becomes smaller as we approach 10. The division between 9 and 10 is the smallest of all.

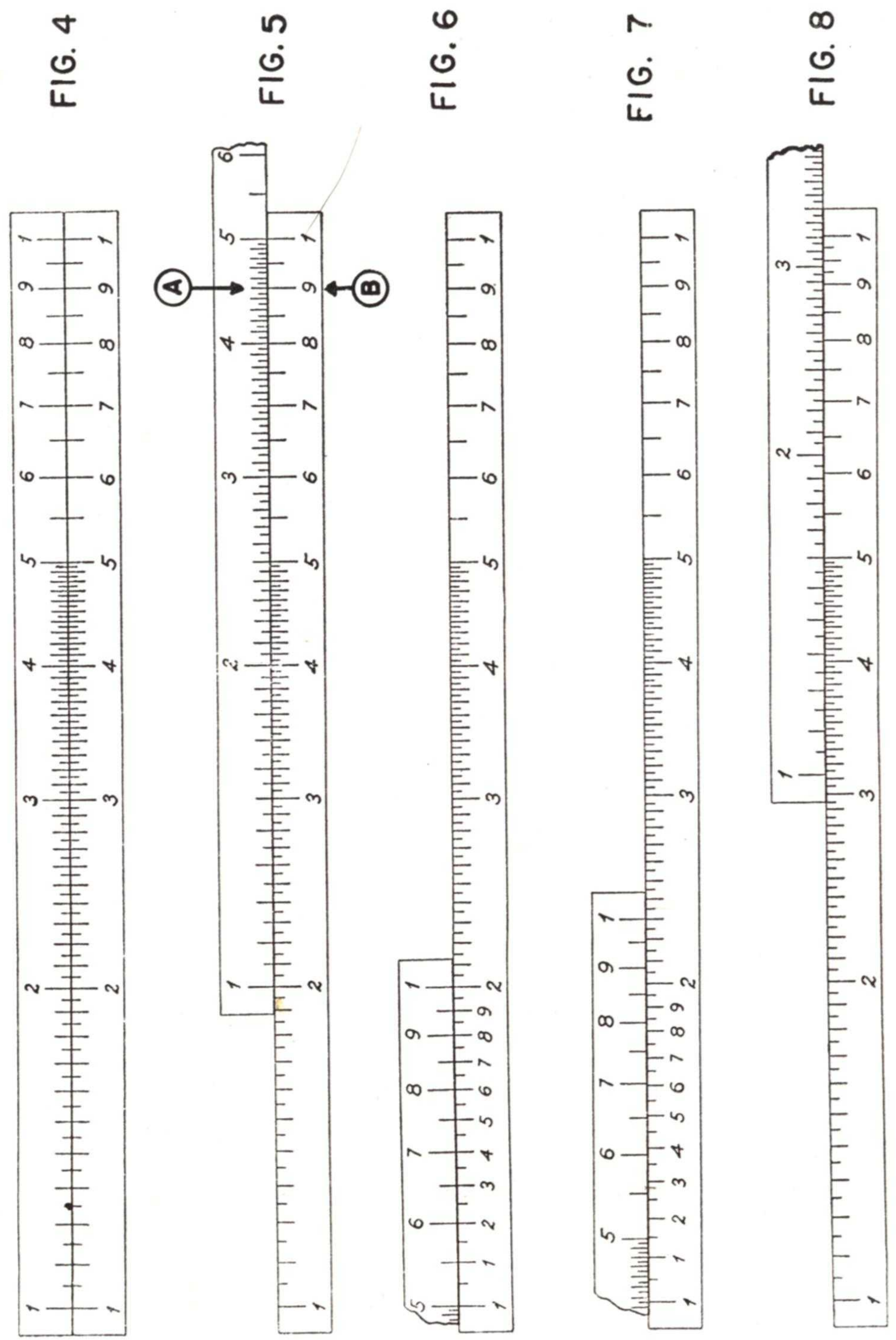
Two logarithmic scales are shown in Fig. 4. They can be considered as examples of two logarithmic scales on a typical slide rule. Now, since we can multiply two numbers by adding their logs, it is obvious that if we set the slide rule so that the logarithmic scales are placed as shown in Fig. 5, we can multiply any number up to 5 by 2 and obtain the product merely by referring to the *multiplicand* in the upper scale and reading the *product* on the *lower scale.*

For instance 4.5 × 2 = 9. We obtain this by setting the multiplier (2 on the lower scale) against 1 on the upper. We now find 4.5 (A) on the upper scale and below it (B) is the answer, 9.

If we want to multiply numbers larger than 5 by 2 we place the scales as shown in Fig. 6 that is, we use the 1 at the other end of the scale, and follow the same procedure, but we must remember that the answer is the number indicated, times 10. For example if you wanted to find 6 × 2, you would set it up as shown and under the 6 you see one and 2 tenths, so the answer is 12.

Stated in a few words, the process of multiplication with a slide rule is as follows: Set the 1 (the index) of the upper scale over the multiplier on the lower scale, locate the multiplicand on the upper scale and read the product on the lower scale directly below the multiplicand.

We can move the upper scale either to the right or to the left, using the left hand 1, or the right hand 1 (10) as the index, depending on which is



more convenient.

By using the various subdivisions we can multiply larger numbers. Suppose we want to multiply 78 by 23. We move the upper scale to the left until the right-hand 1 is over 23 on the lower scale as shown in Fig. 7. (Notice that the largest divisions on the scale are subdivided into tenths. To find 23, first find the 2, then count off three of the subdivisions to find 23. This same point would also be used as 2.3, 23, 230, 2300, etc.) Then locating 78 on the upper scale we read the product on the lower scale and we find it to be 1795. If we multiplied this out by longhand we would get the product as 1794. In practice, 1790 would be close enough, because accuracy to three places is sufficient.

Of course the slide rule does not tell us how many places there will be in the product, nor does it tell us where the decimal point should be. We must determine the number of places and the position of the decimal point by inspection. When multiplying 23 by 78 for example, we can see at a glance that the product will be above 1000 and below 10,000 (because 20 × 80 = 1600). In later paragraphs we shall discuss decimal location, etc., by inspection.

Division by slide rule is just as simple as multiplication. The process is one of subtracting. We use the same scales as in multiplication.

In dividing, we position the upper scale so that the *divisor* is directly *above* the *dividend* and read the *quotient* on the lower scale directly *under* the *index 1*.

Suppose we want to divide 3 into 6. We place the 3 of the upper scale directly *above the 6 of the lower scale.* Then under the index 1 we read the quotient on the lower scale, which is 2. We would divide 300 into 600 or 3,000,000 into 6,000,000 in exactly the same way. Or we could divide 30 into 6,000,000, in which case we would have to determine the number of zeros in the quotient by inspection.

Let us take a slightly more difficult problem such as dividing 969,424 by 31.

We locate the divisor 31 on the upper scale and move it directly above 969 on the lower scale as in Fig. 8. Notice that we disregard the last three figures as insignificant. We now read the quotient directly below the index 1, on the lower scale and find it to be slightly less than 313 (actually 31271). By inspection we know that the quotient must be between 10,000 and 100,000 because 900,000 ÷ 30 = 30,000, therefore we add two zeros to 313 to get 31,300.

If we were dealing with money, of course, this would be too inaccurate. There is too much difference between $31,300 and $31,271.74, but in radio and television and for most practical purposes, the answer given by the slide rule will be close enough.

LOCATING DECIMAL POINTS BY INSPECTION

When using a slide rule, the only way to find out how many places there will be in the answer, or where the decimal point belongs, is by inspection. We shall briefly consider inspection in multiplication and division.

Multiplication. Suppose we want to multiply 3856 × 4.414: Inspection shows that the answer contains five whole figures, for the answer will be a little more than 4 × 3856. Thus, 3856 × 4.414 gives 17,000.

Consider 3856 × 441.4: Think of the number as being multiplied by 4 with the decimal point moved two places to the right. Then the number multiplied by 4 will have five figures, plus two zeros, which will give the answer to 7 places. Thus, 3856 × 441.4 gives 1,700,000.

Consider 3856 × .0004414: Think of the number as being multiplied by 4 with the decimal point moved 4 places to the left. Then the number multiplied by 4 will have five figures, but with the decimal moved 4 places to the left. Thus 3856 × .0004414 equals 1.7000.

Division. Consider the fraction .3856/4414: Think of the denominator 4414 as having the decimal point after the first figure. Then, move the decimal point in the numerator the same number of places in the same direction. In making these mental operations, we think of the denominator as having the decimal point after the first figure, thus: 4.414. Then moving the decimal point in the numerator three places in the same direction, we have .0003856/4.414, and we see that 4 will go into the numerator about .00009. The correct answer is .0000874.

Consider the fraction 38.56/.0004414—We can mentally move the decimal point in both the numerator and the denominator to give us 385600/4.414. We see that 4 will go into the numerator about 90,000 times. The correct answer is 87,400.

PRACTICAL SLIDE RULE CALCULATION

A typical slide rule is shown in Fig. 9. It is known as the Polyphase (Manheim) slide Rule and is manufactured by Keuffel & Esser, 127 Fulton Street, New York City.

You will note that there are two upper logarithmic scales, A on the rule and B on the slider; also two lower scales, C on the slider and D on the rule. The glass with a vertical engraved line through the center is known as the "Cursor," sometimes it is called a "runner" but it is always best to use the correct terminology. A little later we shall see how it is used.

Between the B and C scales on the

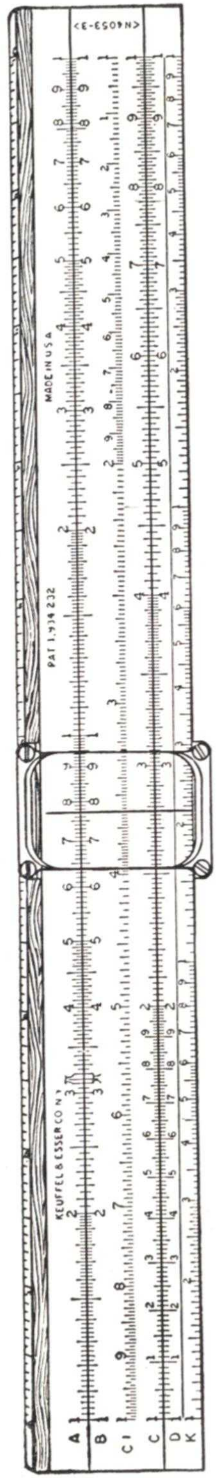


FIG. 9. A typical slide rule made by the Keuffel and Esser Co.

slide there is a "C1" scale, also known as the inverted C scale. Below the D scale we find another scale marked K which is used with the D scale to find cubes and cube roots.

The slider also has three scales on the reverse side. These scales are marked S, L, and T. They are used with the top scales for calculations involving sines, logarithms and tangents.

Suppose we wish to multiply 78 by 23. We shall use the C and D scales. Set 1 on the right-hand end of the C scale above 78 on the D scale; move the cursor so that the cross hair is over 23 on the C scale. Now look down the hair line onto the D scale for the answer 1795.

If we had tried to use the left-hand 1 of scale C as would appear natural at first, reading under 23 on the D scale would be impossible. By using the right-hand 1 of the C scale, we are actually placing a second D scale after the first.

To simplify multiplication, the C1 scale is used. Again let us multiply 78 × 23. Set the cursor on 78 of the D scale, move the slider until 23 on the C1 scale is on the engraved line of the cursor. (Notice that the C1 scale runs from right to left instead of from left to right.) Read the answer below 1 on the C scale—either the right or left hand 1 will indicate the answer. It makes no difference whether 78 or 23 is used on the D scale or which one you multiply by.

Squares and square roots may be found by using the cursor and the rule body only. To find the square of a number, locate the number on the D scale with the cross hair, and read the answer directly on the A scale. For example, setting the cross hair on 4 of the D scale, we find the square is 16 on the A scale. Again, the square of 8 is 64. Note that the numbers could be 8. 80, 800, etc., and the squares would then be 64, 6400, 640,000.

Notice also that the scale is really two log scales exactly alike, and we call the left scale A1 and the right scale A2. As you will remember when finding the square root of a number, we divide the number into groups of two figures each going left and right from the decimal point. For example 25|00, 6|72, 97|40, 5. In determining whether the A1 or the A2 scale is to be used, we consider only the number in the first group, that is, 25, 6, 97, 5. When there are two figures, use A2—when only one, use A1, thus 25 (A2)—6 (A1)—97 (A2)—5 (A1).

To find the square root of 25|00, set the cross hair on 25, on the A2 scale, locate the answer directly below it on the D scale. You will find a 5, and the answer is 50 on the D scale.

If we wanted to find the square root of a decimal number, we would proceed as before to divide our number into groups of figures from the right of the decimal point, thus .00|36.

Rule: If the first figure in the first group after the zeros is a zero, you would use the left-hand scale (A1); if the first figure in the first group after the zeros is a number other than zero, you use the right-hand scale. For example, to find the square root of .0036, you would use the right-hand scale, and the answer would be .06. However, if we wanted to find the square root of .036, we would use the left-hand scale (A1) and the answer would be .19. To find the square root of .36, we would again use the right-hand scale, and the answer would be .6. You can check these answers by squaring them:

$$\begin{array}{r} .06 \\ \times .06 \\ \hline .0036 \end{array} \qquad \begin{array}{r} .19 \\ \times .19 \\ \hline 171 \\ 19 \\ \hline .0361 \end{array} \qquad \begin{array}{r} .6 \\ \times .6 \\ \hline .36 \end{array}$$

To find the cube of a number, set

the cross hair on the number on D, and read the cube directly on K. The cube of 4 is 64; again the cube of 8 is 512.

Notice that the K scale consists of three identical log scales, referred to as K1, K2, K3, reading from left to right. Again we have a rule for determining which one to use when finding cube roots. For numbers greater than 1, begin at the decimal point and mark off the numbers into groups of three figures. If the last group contains one, two, or three figures, we use K1, K2, or K3 respectively. To illustrate, let us take the number 216. This number contains 3 figures, so we use K3. Setting the cursor on 216 on K3, we read 6, directly above it.

If the number is a decimal, we mark off the numbers in threes, beginning from the decimal point and working toward the right, thus: .000|008.

Rule: Again we choose which scale to use by checking the number of zeros in the first group following the groups that are entirely zeros. If the first figure in the group is not a zero, we use K3; if the first figure in the group is a zero, and the *second* figure is not a zero, we use K2; if the first two figures in the group are zeros, we use K1. For example, to find the cube root of .000|008, we use K1 and we get .02. To find the cube root of .000|08, we use K2, and we get .043; to find the cube root of .0008, we use K3, and the answer is .092. For .008, the answer on K1 is .2; for .08, the answer on K2 is .43; for .8, the answer on K3 is .92.

These instructions in general apply to all slide rules. Nevertheless different manufacturers may have different arrangements of scales.

For instance the "Deci. Log Log" made by Pickett and Eckel, Inc., Chicago, does not label the scales A, B, etc., although the C, C1, and D scales are used. For extracting cube roots and square roots, and finding cubes and squares of numbers, individual cube root and square root scales are used and identified by the square and cube root signs.

However, regardless of the individual physical layout, all slide rules operate on the same principle of adding or subtracting logarithms, and once you have mastered the simple operations of division and multiplication on one slide rule, you can apply these principles to any other. Of course, you should always read very carefully, and be sure you understand thoroughly, the individual manufacturer's instructions sheet that comes with every slide rule. When you have once learned to use it, you will find the slide rule a very handy instrument.

ANSWERS TO PROBLEMS

Negative Numbers, page 4: 283 804 164

Multiplying Letters, page 8: $a^2 - b^2$

Subtracting Letters, page 9: (1) $-12a+11b+5c$ (2) $7ab-10d+14y$

Finding Logarithms, page 20: (1) 2.6839 (2) 1.6839 (3) .6839 (4) $\bar{1}.6839$ (5) $\bar{2}.6839$

Multiplying with Logarithms: (a) 54,040 (b) 3153 (c) 65760

RULES FOR LIVING

The late King George V of England formulated six rules for living, each a masterpiece of wisdom in itself, which together form a challenge to every ambitious, red-blooded man.

Teach me to obey the rules of the game.

Teach me never to cry for the moon, never to cry over spilled milk.

Teach me to win if I can; if I cannot win, teach me to be a good loser.

Teach me to distinguish between sentiment and sentimentality—to esteem the first and to despise the second.

Finally, if I must suffer, may I be like a thoroughbred that goes away by himself in order to suffer in silence.

Such are the rules for living set up and followed by a great king; they can be your guide for living. too.