RADIO FORMULAS AND HOW TO USE THEM

REFERENCE TEXT 39X

PIONEER HOME STUDY RADIO SCHOOL

NRI

NATIONAL RADIO INSTITUTE

WASHINGTON, D. C.

ESTABLISHED 1914

ENGINEERING DATA FOR RADIO-TRICIANS

The primary purpose of this Course is to prepare you to install, operate, adjust and service radio, television and electronic control apparatus. Mathematical formulas are relatively unimportant in accomplishing this purpose, although some may find that a knowledge of formulas speeds up their work.

Occasionally, however, a Radiotrician finds it necessary to design a particular piece of equipment; he must then be able to predict beforehand how the unit will perform and must be able to compute the electrical sizes required to give the desired results. To those who want to design special apparatus with the least amount of experience, this book of formulas will be of great benefit. It is truly a valuable reference book, for in it has been combined the essential design data for many different devices.

For the present it will be sufficient if you simply go over the table of contents to find out exactly what is in the book, then spend an hour or so glancing at the material in it which interests you. After this, place the book in your reference library, where it can serve you long after you have graduated from this Course.

NATIONAL RADIO INSTITUTE

WASHINGTON, D. C.

1950 Edition

A LESSON TEXT OF THE N. R. I. COURSE WHICH TRAINS YOU TO BECOME A RADIOTRICIAN & TELETRICIAN
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RADIO FORMULAS AND HOW TO USE THEM

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How To Use Formulas

WHAT IS A FORMULA?

Before we get too involved in the use, manipulation and the type of formulas to be presented in this text let us make a few things clear. This text is only a reference book and contains simple, every day formulas as well as those of particular interest to the more advanced student. Keep it like a dictionary, referring to it only when the need for a formula presents itself. Only the formulas which are of use in the more practical phases of radio are given. Read the rest of the text so that you will know what this text contains and where to look for a specific formula when you need it.

The dictionary, which every student and radiotrician should learn to refer to, says that "any general fact, law or principle expressed in algebraic symbols" may be considered a formula. We have so far in our study of radio met many formulas which bear out the above definition. Let us consider how a fact, a rule or a principle may be expressed in terms of algebraic symbols.

Everyone knows that a train running at a uniform speed, for example constantly at the rate of 30 miles an hour, will travel a longer distance in 30 minutes than in 5 minutes. Knowing the relationship of these factors we can form a general statement as follows. The distance covered is equal to the uniform rate of travel times the time running. In fact this is so general that it applies to automobiles, runners, airplanes or anything that is in uniform motion. It may even apply to a sound or radio wave which is known to travel respectively at 1,089 feet per second and 186,000 miles per second. To set this up in algebraic form or notation, as it is called, let us say that the letter D will represent the distance traveled, that V will represent the uniform rate of travel (velocity) and t the time during which the motion we are concerned with takes place. Therefore the above statement regarding distance may be expressed in algebraic terms as

$$D = V \times \qquad (1)$$

Observe that on the left-hand side of this formula we have D the factor we are to compute when we know the factors V and t, which are on the right-hand side. This algebraic statement of a fact is essentially an expression of equality, since what is on the left-hand side is equal to what is on the right-hand side—an equation. We say as much when the symbol (=) is used for *equals*.

Now there is a slight practical difference between an equation and a formula, which may be worth knowing. If the factor on the left-hand side is the unknown and all the factors on the right-hand side are known, we have a formula. If both the right and left-hand sides contain unknown factors, we have an equation. We shall shortly see how an equation may be transformed into a formula. We will now stress the fact that an equation is useless for purposes of calculation of the unknown unless all factors except the unknown are given or assumed to have some definite value.

Let us go back to our original idea about a moving train. We said that it travels 30 miles an hour. We may be interested in knowing how far that train will travel in 16 minutes. Note that in one case we have used the hour as the time unit and in the second case we have used minutes. In any formula where the factors of time, distance, area, etc., are employed we must be careful to use the same dimensions or units. Never mix hours, minutes and seconds; never mix miles, feet and inches unless the legend associated with the formula permits such an assumption. You would not say that 4 cows and 3 horses make 7 cows.

Thus, in order to carry out the principle of using proper units in the problem just given, we must either convert the rate of travel into miles per minute or the time into hours. Using the first scheme, we say that 30 miles an hour is the same as ½ mile per minute, simply because there are 60 minutes in an hour and 30 ÷ 60 is ½. You will get along much better in practical work if you express numbers in terms of decimals. So we would say .5 instead of ½. Everyone knows that if a train moves .5 mile per minute that in 16 minutes it will travel 8 miles. But if we use formula (1), we would go about it in this way. We would say that V equals .5, and t equals 16. Substituting in the formula we get

$$D = .5 \times 16$$
$$D = 8$$

Now let us consider other types of formulas.

There is a law or a principle in physics that stipulates that energy can neither be created nor destroyed. For example, if you send an electric current through a resistor, the electrical energy is transformed entirely into heat energy. We know from a study of electricity that the electrical energy is equal to the power multiplied by the time during which the power is supplied. In algebraic notation let us call W the energy, P the power and t the time, which with the above permits us to show that

$$W = Pt \tag{2}$$

Note that the multiplication notation (×) is omitted in this formula. If P and t denote separate factors, it is customary when algebraic symbols are used to omit the sign of multiplication.

We also know that the power absorbed by a resistor is equal to the square of the current multiplied by the ohmic value of the resistance, and of course we are all familiar with the formula:

$$P = I^2R \tag{3}$$

We may now say that the energy absorbed by the resistor is equal to the product of the square of the current, the ohmic value of the resistor and the time. As a formula we say:

$$W = I^2Rt \tag{4}$$

Formula (4) means little to us until we know the dimensions of W, I, R and t, so as a useful formula we should say

The Formula→ $W = I^2Rt$ (4a)

The Legend→ Where W is in watt-seconds or joules
I is in amperes
R is in ohms
t is in seconds

Although the expert from long experience knows what to substitute in formula (4), the average man needs the information or legend given with the formula as in (4a). We must understand the dimension of the algebraic symbols if we want to make practical use of a formula.

RADIO USES FOR FORMULAS

Formulas are extremely helpful in service work. This does not mean that you cannot service without using formulas. For example, a C bias resistor burns out and it must be replaced. What replacement resistor should you use? If the service circuit diagram tells you the resistance value and the power rating, trying to figure out the proper resistor to use would be entirely unnecessary. In some cases, however, only the resistance value may be given. Shall you use a 1, 2, 5, 10, 25, or a 50 watt resistor? You know that the higher the rating the more costly will be the replacement resistor. Experience may tell you what power rating it should have. Substitution in a simple formula will remove all doubt. Again, maybe neither the value of the resistance nor the power rating is known. You may insert a variable resistor shunted with a voltmeter and adjust the resistor until you get the correct bias and then guess at the power rating. Simple calculation will eliminate this and even remove the errors of measurement.

Suppose you make a point to point resistance test. The chances are that you will not be told the net resistance between the two test points. If two or more devices are in series, you may simply add their respective ohmic values. Suppose some device is shunted by a resistor. What then? You must know how to compute the total resistance or the test value will be useless to you.

Should you decide to use a resistance-capacitance filter to buck out hum, you may juggle and change resistors and condensers until you get the right combination. If you compute the correct values from a formula, you will have accomplished a considerable saving in time.

If you want to extend the range of a milliammeter or a voltmeter, you will find that calculations will make the extension easy. Suppose you want to build an oscillator for service work, using a variable condenser that you have on hand. You may guess at a coil, then add or take off turns until by test you hit the right range. No doubt you may have to when you calculate the correct coil. The chances are good, though, that you will never need to add turns and probably will have to take off only a few turns if you start with calculations.

Example after example could be given to prove the helpfulness of computation with formulas. You may or may not find them useful or a time saver. People differ in this respect.

When we come to radio design, we find formulas an absolute need. Building a receiver or transmitter from blueprints or a kit is not design. It is merely assembly. What the value of a resistor, a coil or a condenser should be when starting from "scratch" is a problem of design in which formulas play an important part. You can't use formulas blindly, for the theory of the circuit and the effect desired has a lot to do with choosing the correct formula. That comes from your study and experience in radio.

NOT ALL TERMS ARE VARIABLES

Now let us look at a formula in more detail. We have by custom placed the unknown factor on the left-hand side of the formula, while on the right may be placed a simple or complex algebraic arrangement of known factors. These are called variables. To distinguish them, the known factors are referred to as independent variables because we may assign to them any desired value. The unknown factor is referred to as the dependent variable because it will vary as we vary the terms on the right-hand side, and its value will depend on what values are assigned to the independent variables. A simple example:

$$X_L = 6.28\, fL \tag{1}$$

Where X_L is in ohms when
f is in cycles per second
and L is in henries

Here f and L are the independent variables for f the frequency may be 60, 120, 5,000, or 1,000,000 cycles per second. L may be 30, 2, or .002 henries. X_L will vary as we carry f and L, and its value in a particular case will depend on what values we assign to f and L. For example, if f is 60 c.p.s. and L is 10 henries, X_L according to the formula will be:

$$X_L = 6.28 \times 60 \times 10$$
$$= 3768 \text{ ohms.}$$

What about the number 6.28 in the formula? First of all we know it is a number that does not vary as we change f and L, under any condition. For that reason it is often called a constant. To be able to talk intelligently about these constants we must know how formulas are obtained.

Most of the formulas you will find in this reference text are the result of mathematical deduction. Mathematicians, fortified with such basic truths or laws as Ohm's Law, Kirchoff's Law, the law of conservation of energy, derive by mathematical manipulations, equations or formulas. As they try to establish practical facts from basic laws they obtain formulas that are extremely helpful to scientists, engineers and practical technicians.

We will not go into the derivation of formulas from basic facts. In fact, only a few men bent on research work and equipped with a knowledge of higher mathematics and practical physics can attempt such a procedure. Let us take what these capable men have provided and use their results as we see fit. In other words, let us stick to our specialty—for we live in a world of specialists.

Now what does all this have to do with the constant 6.28? The fact is that formula (1) could have been written as

$$X_L = 2\pi fL \tag{2}$$

Apparently 6.28 must equal the expression 2π. The notation π (pronounced pie) is a geometric notation to express the ratio of the circumference to the diameter of a circle. It has a value of 3.14159 plus a string of numbers. Some one worked it out to about 600 decimal places. For radio purposes 3.14 is good enough. Therefore 2×3.14 equals 6.28. Now how did the number 2 and π get into the formula?

Without getting into deep mathematical discussion let us say that constants like these get into formulas as the result of expressing electrical ideas in geometric terms —circles, arcs and spheres. Thus 2π in formula (2) may be the result of expressing the ratio of the circumference to the radius of a circle. You will find a number of mathematical constants in radio formulas, particularly:

$\pi = 3.14$	$4\pi^2 = 39.5$
$1/\pi = 0.318$	$\sqrt{\pi} = 1.77$
$2\pi = 6.28$	$\epsilon = 2.72$*
$\pi^2 = 9.87$	

* ϵ is pronounced epsilon.

We have considered one way in which a constant may get into a formula. Let us consider another important way. We said that the electrical energy absorbed by a resistor of value R ohms when a current I amperes flows through it for t seconds will be given by the formula:

$$W = I^2Rt \text{ watt-seconds or joules}$$

But we know that this energy is transferred to heat and, if we have any knowledge of the branch of physics referred to as heat, we will know that heat energy is expressed in calories. One calorie is the energy required to raise the temperature of one cubic centimeter of water 1 degree Centigrade. Obviously, if we were a station operator where water cooled tubes were used, we would be interested in knowing the formula involving calories. By the law of conservation of energy we know that electrical energy is transformed into heat energy and there must be some number or constant that will express joules in calories. We show that in algebraic form thus:

$$W = KI^2Rt \text{ calories} \tag{3}$$

We call the letter K the constant of proportionality; in fact formulas are full of them. By experimental evidence K in formula (3) is 0.24 and we rewrite the formula as:

$$W = 0.24\, I^2Rt \text{ calories} \tag{4}$$

Here is another way of showing how the constant in a formula may apparently vary. You are all familiar with the resonance formula:

$$= \frac{1}{2\pi\sqrt{LC}} \tag{5}$$

Where f is in cycles per second
when L is in henries
and C is in farads

But the farad is a very large condenser unit value never met with in practical problems. The microfarad is a more practical dimension. We may express formula (5) as follows: Where f is in cycles, L is in henries, and C is in microfarads, thus:

$$= \frac{159.2}{\sqrt{LC}} \text{ c.p.s.} \qquad (6)$$

For very high frequencies, even the henry is too large as a dimension, so formula (6) may be expressed thus

$$= \frac{159{,}200}{\sqrt{LC}} \qquad (7)$$

Where f is in cycles
when L is in microhenries
and C is in microfarads

VISUALIZING FORMULAS

In your study of radio you must have noticed that a formula had other uses than for the purpose of computing the dependent variable. Formulas were introduced to give you some idea of the relationships involved in certain electrical phenomena. Take a simple case of heat dissipation in a resistor. The power loss is given by the formula:

$$P = I^2R \qquad (1)$$

Where P is in watts
I is in amperes
R is in ohms

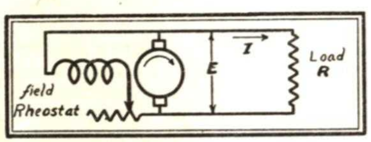

Fig. 1

Of course, in a simple circuit like Fig. 1, consisting of a resistor connected to a variable voltage generator, we may change R, or vary I by changing the voltage. Suppose we double the ohmic value of the load resistor R, keeping I constant by adjusting the generated voltage. Formula (1) tells us that the power loss doubles. On the other hand, suppose we double the current, keeping the load resistor constant, then formula (1) tells us that the power is increased four times, or, as the expert puts it, "as the square of the current." You can only get the full meaning of all this by substituting numbers for I and R in the formula. For example, let $R = 2$ ohms, let I at one time be 2 amperes, 4 amperes, 8 amperes, and etc. Figure out the power loss by means of the formula.

When $I = 2 : P = 2 \times 2 \times 2 = 8$ watts
$I = 4 : P = 4 \times 4 \times 2 = 32$ watts
$I = 8 : P = 8 \times 8 \times 2 = 128$ watts
$I = 16 : P = 16 \times 16 \times 2 = 512$ watts

Such a series of substitutions have the effect of portraying the formula. We learn that, when the current and resistance are increased, the effect of current on power loss is much more than the effect of resistance. This is what experts refer to as visualizing the formula.

So important is the visualizing of formulas that in the discussion of radio theory you will find that formulas or, to be exact, equations are presented with the sole purpose of showing how the dependent variable is affected by the independent variable. For example in the discussion of the force which moves the cone in a moving coil loudspeaker unit we may say that

$$F = KBNI \qquad (2)$$

Where F is the force
B the flux density
N the number of turns in the voice coil
I the current through the coil
K the proportionality constant.

As long as the value of K is unknown and the dimensions of B, N, I, and F are not given, formula (2) has only the power to help us visualize how F the force is affected by B, N, or I. We are told that increasing the flux density in the air gap (increasing the field current up to saturation) produces more force. Likewise, increasing the voice coil turns or current has the same effect. Even though the formula has no use in calculation, we may in design find that such a formula is valuable. After the speaker has been made, we may find that the force produced is too great. This formula tells us that if the flux or the coil turns are reduced the force will be proportionately reduced.

The constant K may be determined experimentally, if we set up a representative moving coil system and measure B, N, I and F. As we will see shortly, formula (2) may be rearranged as

$$K = \frac{F}{BNI} \qquad (3)$$

By substituting the values of B, N, I and F in this formula, we may compute K. As long as we do not alter the geometry of the system; that is, as long as we use in formula (2) the same dimensions that were used in formula (3) to compute K, we may assume that the value of K so computed will give us the correct result for F upon substituting specific values for B, N and I.

There is another way of getting K. We

have a more basic formula for force derived by mathematical physics namely:

$$F = BlI \qquad (4)$$

Where F is the force in dynes
B is flux per square centimeter
l is length in cm. of wire perpendicular to flux
I is the current in abamperes
(10 amperes = 1 abampere)

But note that l may be replaced by $l'N$, where l' is the average length of one turn of the voice coil in centimeters and N is the total number of turns. Comparing the formulas (2) and (4), it is obvious that l' and K are equal when the units given in formula (4) are used. However, if it is desired to express the force in more practical units, such as pound, K must be changed to include the factor of proportionality necessary for converting dynes to pounds.

But all we have said regarding K does not alter the use of formula (2) for purposes of visualizing, even if K is not known.

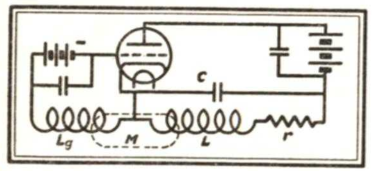

Fig. 2

Let us take one more example of the power of a formula to help visualize circuit conditions. Figure 2 is a simple tuned plate oscillator, using a tube having μ as the amplification factor and G_m as the mutual conductance at operating potentials. We say that oscillation will begin if the following relation is true:

$$M \geqq \frac{Cr}{G_m} + \frac{L}{\mu}$$

Where M is in henries
r is in ohms
C is in farads
G_m is in mhos
L is in henries
μ is a number

The notation $\geqq$ means "equal to or greater than," the value computed after the terms on the right side have been replaced by values and the total evaluated. This formula allows us to visualize the following facts: M, the mutual conductance between L_g and L, may be less for a tube having large values of G_m and μ. A large load r, which may be the resonant circuit resistance, with or without an applied load, calls for a larger coupling, M. It tells us that if L is large C may be small without altering the condition for oscillation. Of course this formula tells us nothing about the oscillator once it is in operation. It merely helps guide the design of an oscillator that will at least start to work.

EXPRESSING FORMULAS GRAPHICALLY

The previous section brings us face to face with the fact that we may go a step further in visualizing formulas. We may express the formula in picture form, generally called a curve or a graph. If drawn roughly from inspection of the formula, it is usually done so to convey generally the effects that the independent variables have on the dependent variable. Experts can look at a formula and roughly draw a curve showing the desired relation between known and unknown factors.

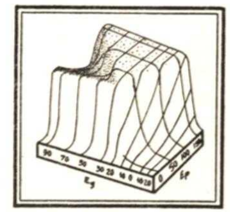

Fig. 3

Graphs and curves are not new to you, as you have constantly met them in your study of radio. But here let us investigate curves a little more critically.

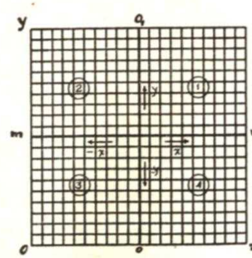

Fig. 4

As far as the practical man is concerned, a formula may be represented as lines on paper or, as we say, in two dimensions. To be sure, we may represent formulas as a solid or curved surface, often referred to as three dimensional representations. Perhaps you have seen clay models of formulas as in Fig. 3. The latter is particularly valuable when you wish to portray how the dependent variable depends on two independent variables. We will shortly see how all such formulas may be replaced by a representation on graph paper.

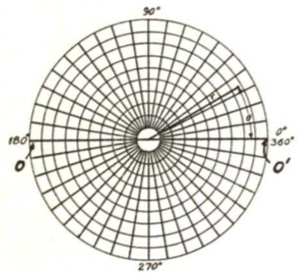

Fig. 5

In representing formulas we encounter several types of graph or plotting papers as represented by Figs. 4, 5, and 6. Figure 4 is referred to as rectangular coordinate paper, the vertical lines to the right and left of oO_1, representing* various values of the independent variable, or x, as it is often called in algebra. The horizontal line above and below the line mm_1 represents the value of the dependent variable, or y, as we often call it in algebra. In such a representation the vertical and horizontal lines are uniformly spaced and may represent any desired value of the factor: 2 feet, 2 henries, 2 microfarads, 2 micro-microfarads; or 4, 6, 10, 20 feet. Usually you will find, on rectangular coordinate graph paper, bold horizontal and vertical lines each separated by light lines dividing the bold lines in 5 or 10 equal spaces. For reason of simplicity in plotting and reading curves it is always well to have each major division represent some multiple or submultiple of 10—10, 20, 1, .01, etc. No doubt you have recognized all this

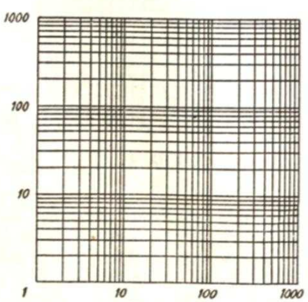

Fig. 6

* Lines oO_1 and mm_1 are drawn in if one wishes to plot in four quadrants, that is when there are + and − values of x and y. In most cases plotting in one quadrant is sufficient, as will be shown.

from the curves presented in the regular texts.

When you have a formula involving one independent and one dependent variable you can present it on a graph like Fig. 7 by calculating the values of y (the dependent

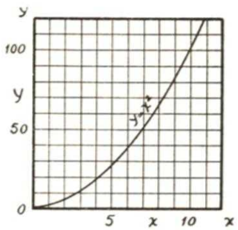

Fig. 7

variable) for various values of x (the independent variable). Spot the vertical line corresponding to x, and the horizontal line corresponding to y. At their intersection place a dot, a small circle or a cross. After locating several intersections, connect the markings with a smooth curve, using preferably a "french curve" (see Fig. 8) available at any drafting supply house. When

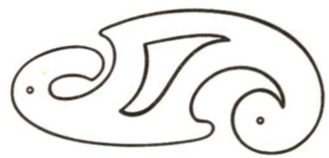

Fig. 8

drawing a curve with figures computed from a formula, you should be able to draw a smooth curve through all points. Although Fig. 4 shows four quadrants so as to represent + and — values of x and y, we rarely have to draw such curves, as negative values may be represented in a single quadrant, merely by noting the fact on the curve. As shown in Fig 7, ox then represents x and oy represents y.

Figure 5 shows another form of graph paper—polar coordinates. In rectangular coordinates any point on the surface of the paper can be referred to by means of two reference coordinates, the x dimension and the y dimension. In polar coordinates, likewise, any point can be referred to or located with two reference coordinates, but one is an angle and the other a linear dimension or simply a length. Polar coordinate paper is generally used to represent formulas where an angle is involved and you want to retain the physical significance of the angle. For example: representing the shape of a straight line frequency condenser, or the special cut plate used in the oscillator of a superheterodyne receiver, or the field intensity around a transmitting antenna. In such cases the angle θ is the independent variable and the radius r, the dependent variable. Note that this graph paper is laid out so it is easy to spot any angle from 0 to 360 degrees and it is easy to assign any value to the various circles. A point at a distance r from the center O measured at an angle θ with respect to the horizontal reference line OO' establishes one of the points on the curve to be drawn.

Figure 6 illustrates the log-log plotting paper and is quite valuable in representing formulas involving logarithms of the independent and dependent variables. Log plotting papers are also made so only one of the rulings is spaced according to logarithms called semi-log paper. You will find a large number of formulas, which are best visualized by plotting on log-log or semi-log paper.

Now why is a picture of a formula so valuable? First you have a clearer insight to the formula. You can tell whether one factor changes faster or slower with respect to the other, observe if saturation is realized, whether there are maximum and minimum values, and how many. Graphs if carefully drawn may replace in practical work subsequent calculations using the formula. Approximate but valuable results are quickly obtained.

Going a step further, suppose we consider the formula for determining the impedance of the circuit shown in Fig. 9.

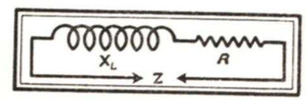

Fig. 9

The formula is:

$$Z = \sqrt{R^2 + X_L^2} \qquad (1)$$

Where Z is in ohms
R is in ohms
X_L is in ohms.

Here we have a formula with two independent variables R and X_L, and one dependent variable Z. Practical radio men want a simple picture of this formula, considering the fact that both R and X_L may vary. Suppose that R and X_L may each be any value from 0 to 100 ohms. We may start by saying that R is 0 and compute Z for various values of X_L from the simplified formula

$$Z = \sqrt{0^2 + X_L^2} = \sqrt{X_L^2} = X_L \qquad (2)$$

Obviously Z will equal X_L.

We may next assume R equal to 10, 20, 30, etc., ohms and compute from formula (1) the values of Z when X_L is 10, 20, 30, etc., ohms, from the formulas

$$Z = \sqrt{10^2 + X_L^2} = \sqrt{100 + X_L^2} \qquad (3)$$

$$Z = \sqrt{20^2 + X_L^2} = \sqrt{400 + X_L^2} \qquad (4)$$

$$Z = \sqrt{30^2 + X_L^2} = \sqrt{900 + X_L^2} \qquad (5)$$

....................etc.

Note that we have held one independent variable fixed while we substituted for the other variable. Thus we may have 11 curves to represent formula (1), by assuming R in one case to be 0 ohms, and in other cases to be 10, 20, 30, 40, 50, 60, 70, 80, 90 and 100 ohms. Plotted we get a group of curves as shown in Fig. 10 which are called a family of curves.

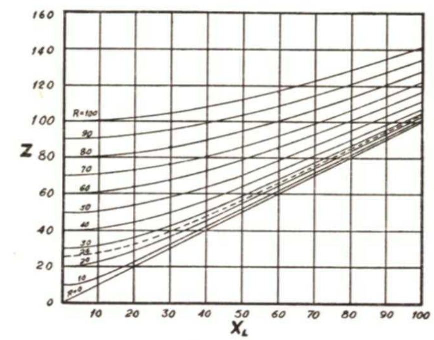

Fig. 10

Note that in such a representation each curve of the family is marked with the value that was assumed for the independent variable held fixed in computing that curve.

If desired, a second family of curves similar to Fig. 10 may be obtained to show how Z varies with R for a series of fixed values assigned to X_L. This is often done, particularly in the study of relationships somewhat more complex than that represented by formula (1).

In this simple case this is not necessary since we may use the family of curves of Fig. 10 to find Z for any of the value of R or X_L within the range of values specified. For example, if you wish to assume a value of 26 ohms for R, we can imagine a curve between $R = 20$ and $R = 30$ (as shown dotted) and thus find Z for any value of X_L between 0 and 100 ohms.

EMPIRICAL CURVES

Quite often we start with a curve or a family of curves and then derive the formula. It is then called an *empirical* formula, meaning a formula derived from observation or experience. Compare this with the formula derived by mathematical deduction. We should remember that a formula derived mathematically is checked by experiment by comparing the curve drawn from the formula, with the empirical curve.

There are many cases where the only solution to the problem is the result of experiment. If the phenomena is one where a formula would be valuable, one may be derived from a curve in which the results obtained experimentally are plotted with precision. Here is a typical case. In the manufacture of many of the basic products for radio equipment, high temperature furnaces are used. The temperature may be measured by inserting a platinum wire resistor and measuring its resistance. Each value of resistance in turn represents a definite temperature. Let us see how the corresponding values of temperature can be determined.

First of all, there is the simple fact that most metals change their resistance with temperature. By placing the resistor in a chamber in which the temperature, *t*, may be varied in known steps and then measuring the resistance at these representative temperatures enough figures are obtained to draw a curve, similar to Fig. 11. If you use the same resistor and the same curve, you have a temperature measuring device.

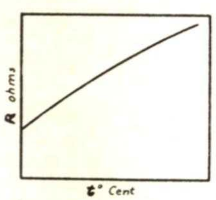

Fig. 11

Going a step further we find that the type of curve shown in Fig. 11, is typical of all metal resistors and may be expressed as an empirical formula:

$$R_t = R_o\,(1 + at + bt^2) \qquad (1)$$

Where R_t is the resistance in ohms at
t the temperature in centigrade
R_o is the resistance at 0 degrees Centigrade
and a and b are constants depending on the metal used.

By carefully analyzing the experimental curve we would find that for platinum wire, $a = +.00392$ and $b = -.000000588$, in which case formula (2) becomes

$$R_t = R_o\,(1 + .00392t - .000000588t^2) \qquad (2)$$

Here is another type of constant found in formulas. The values a and b are found mathematically from the experimental curve by a complicated process which we need not consider.

Formula (2) also tells us that in practical cases the term ($-.000000588t^2$) is negligible in comparison with the first term ($.00392t$) if the temperature t is not much higher than the reference temperature, 0 degrees Centigrade. We rarely expect parts in radio equipment to be over 50° C. If we use this as a limit, we may compare the two terms by substituting 50 for t. Thus:

$$+at = .00392 \times 50 = +.196$$
$$-bt^2 = .000000588 \times 50 \times 50 = -.00147$$

So if we neglect the second term we may have an error less than 1 per cent—which for practical purposes is quite all right. But, in the case of the furnace at 1000° C., b is a very important factor. Thus in low temperature work formula (1) reduces to:

$$R_t = R_o\,(1 + at) \qquad (3)$$

The idea of neglecting terms in a formula is very important and is used time and time again in radio work. Most solutions to radio problems can only be relied upon to about 5 per cent. So why complicate the work with useless computation? When terms in a formula have negligible effect on the answer, they should be neglected. Only experience or trial can guide you in this phase of formula simplification.

In practical radio the empirical curve is far more important than the resultant empirical equation or formula. The E_g-I_p, E_p-I_p, E_g-I_g, fidelity, sensitivity, selectivity, field radiation, and magnetic curves are only a few of the empirical curves that are used directly and never interpreted into a formula. Curves like formulas are essential for our purpose, in that they give practical information. If curves are simpler to get, are more direct and do the job, why try to make a formula out of them? Especially where the formula would not apply in all cases. Radio men do not try. They use the curves when it is to their advantage to do so.

FORMULAS INVOLVING CURVES

You will find a large number of radio formulas where the right hand terms include some factor whose value must be determined from a graph or perhaps a table. This table or curve may be the result of experiment or it may be the result of expressing complicated algebraic expressions in their simplest terms.

The most notable example of such a case is the computation of inductance from the geometry of the coil. The inductance of a round solenoid coil may be given by the formula

$$L = FdN^2 \qquad (1)$$

Where L is in microhenries
N is the number of turns
d is the diameter of the coil in centimeters or inches, depending on whether measurements are made in inches or centimeters
and F is a factor determined from a curve, see Fig. 12, depending on the ratio of the length of the coil to its diameter

In using formula (1) we may either have a coil with a definite number of turns and with a known coil length and diameter which permits us to compute L; or we would from trial and error try various N turns, l lengths and d diameters until we found a combination that would give the desired L.

From Fig. 12 we determine F for the ratio of l/d for each combination and substitute the value

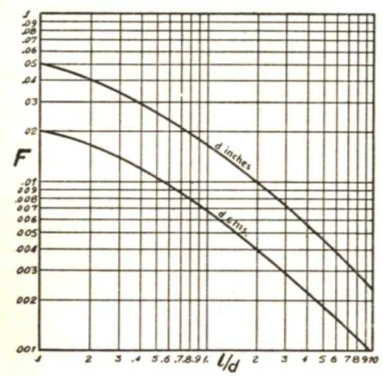

Fig. 12

in formula (1). In this case F is merely a constant that could be computed from l and d by a complex algebraic expression. The curve simplifies the problem of practical computation.

EVALUATING THE UNKNOWN

By now we know that a formula has on the left hand side the factor or variable that we want to compute. On the right hand side, expressed in algebraic manner, are the constants and independent variables that are known or quickly found from tables or curves. To evaluate the unknown we merely substitute on the right hand side the values for the algebraic notation and by mathematical operation derive the final single number or value. We may look on this as sort of a mill.

It is highly important that this reduction process be as systematic as possible, otherwise you will get into a tangle. There is only one way of developing the technique of manipulating algebraic reduction—by working at it. Take a simple formula, for example, the case of an inductive reactance, a capacitive reactance and a resistance all in series, as shown in Fig. 13.

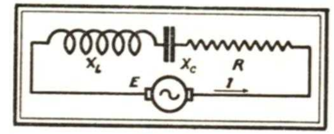

Fig. 13

If an A.C. voltage E is connected to this circuit, the current I flowing in the circuit will be given by the formula:

$$I = \frac{E}{\sqrt{R^2 + (X_L - X_c)^2}} \qquad (1)$$

Where E is in volts
R, X_L and X_c are in ohms
I is in amperes

We know that the right hand terms will reduce to a single valued number if we know the value of E, R, X_L and X_c and substitute them into the algebraic expression. Suppose $R = 2$; $X_L = 5$; $X_c = 2$; and $E = 10$. How would you go about reducing the right hand terms to a number? First of all, we substitute in formula (1) the number for the letters, thus:

$$I = \frac{10}{\sqrt{2^2 + (5-2)^2}} \qquad (2)$$

Our next problem is to get rid of the complex expression $\sqrt{2^2 + (5-2)^2}$. But let us do this in steps. First we know that $(5-2)$ equals 3; therefore $(5-2)^2$ must equal 3^2. So we obtain:

$$I = \frac{10}{\sqrt{2^2 + 3^2}} \qquad (3)$$

We know that 2^2 equals 4, and 3^2 equals 9, so as the next step in simplifying expression (3) we get

$$I = \frac{10}{\sqrt{4+9}} = \frac{10}{\sqrt{13}} \qquad (4)$$

Our next step is to evaluate $\sqrt{13}$. You may use the long method as taught in grammar school, or use the slide rule or logarithms as explained in a previous text. We will find that $\sqrt{13}$ equals approximately

3.60. Therefore, the next steps in evaluating the value of I when E is 10, R is 2, X_L is 5, and X_c is 3, follows:

$$I = \frac{10}{3.60} \qquad (5)$$

and by division:

$$I = 2.77 \text{ amperes.} \qquad (6)$$

You will find the evaluation of the unknown simple and quick if you follow a systematic method and realize the importance of certain algebraic notations. Thus:

ab means a multiplied by b
$a \div b$; a/b means a divided by b
$a + b$ means a added to b
$a - b$ means b subtracted from a
a^2 means a multiplied by itself (aa)
a^3 means a multiplied by itself twice (aaa)
$\sqrt[2]{a}$; $a^{1/2}$ means the square root of a
$\sqrt[3]{a}$; $a^{1/3}$ means the cube root of a
$\sqrt[3]{a^2}$; $a^{2/3}$ means the cube root of the square of a
$2.72^{1.32}$ means the 1.32 power of 2.72

Furthermore, you may find expressions like $(4^2 + 2)\ 1.25$, the brackets ( ) indicating that you should first evaluate the terms within the bracket before multiplying by 1.25. Again you may find expressions like:

$$\sqrt{[(4^2 + 2)\ 1.25 + 29.3]\ 6.28}$$

which indicates that the term within the parenthesis ( ) is evaluated first; then the result is multiplied by 1.25, then this result added to 29.3, before multiplying by 6.28. Now you may find the square root of the resultant number.

In the case of fractions, always reduce the numerator and denominator to a single valued number before dividing.

SIGNIFICANT FIGURES

The number of significant figures to be used when substituting numerical values into a formula, and the number to retain in the final answer is important. Starting with more significant figures than are required is a waste of time and effort and does not yield a more precise solution. Don't overlook this fact. The subject, significant figures, is not new to you.

Precision of measurements is the important factor in determining how many significant figures you shall start with and retain in the answer. Let us take a simple example. Assume that we loaded a generator with a 3 ohm resistor and then measured the terminal voltage as 10 volts. Then by Ohms law, the current:

$$I = E/R = 10/3 \qquad (1)$$

Strictly speaking, $10 \div 3$ equals 3.33333333+ etc., indefinitely or until we get tired of writing the numeral 3. Suppose an 0 to 5 ampere meter was inserted in the circuit and for the sake of simplicity this meter had negligible resistance. What current value do you think you would read? If the meter is one of those used in ordinary radio work, the meter maker tells you beforehand not to rely on it to more than 2%. So with this as a start you may read 10/3 plus 2% or 10/3 minus 2%, which means that if everything else were perfect you may read any value between 3.26 and 3.39 amperes. The next question is in reading the meter scale as close as the latter values. The fact remains that you may read any value between 3.2 and 3.4 amperes. Common sense tells us that 3.3 amperes is a more reasonable answer than 3.33333 + . . . etc.

In this simple problem there are other reasons why too many significant figures may be in error. In the first place, with what precision did we measure voltage, and measure the resistance? If you used an ordinary voltmeter calibrated to within 2%, you may have read 10 volts but could not rely on the value as being correct. The actual value may be between 9.8 and 10.2 volts. Likewise the resistance may be measured as 3.0 ohms, but may be larger or smaller than this value, depending on how precise is the measuring equipment. When you place a voltmeter in the circuit you disturb the circuit and 10 volts may be slightly low.

The upshot of the whole matter is, take a practical attitude towards significant figures. Use reliable measuring equipment and substitute in the formula the numbers that are obtained from measurements. Other figures —that is, constants—should not have any more significant figures. Here is an example:

$$X_L = 2\pi fL \text{ (ohms)} \qquad (2)$$

Where f is in c. p. s. and
L is in henries

We may assume that the frequency of the supply is 291 c.p.s. and we measure L to be 6.2 henries. The value of π is 3.14159 to 6 significant figures. It will be perfectly safe to use the value 3.14. Thus:

$$X_L = 2 \times 3.14 \times 291 \times 6.2 = ?$$

If you follow the long multiplication method, you will get the absurd answer of $X_L = 11330.376$; if you follow the short method used by engineers, you will get $X_L = 11330$; if you use a 10 inch slide rule, you will get $X_L = 11330$.

Slide rule calculations are as close as you will need to compute on a sensible basis.

That is why every engineer and technician uses a slide rule.

COMPUTATION CHARTS AND TABLES

Magazines, texts and articles intended for the average technician often have special tables for finding squares and square roots, cubes and cube roots. Countless charts have been prepared to find the value of resistors in parallel, resonant frequencies of a coil and condenser combination, and other similar values. Of course they are time savers, and you may use them if you wish if they are available.

We feel that such schemes defeat the desired purpose of formulas. If you get into the habit of using charts and tables you develop mental laziness and fail to use formulas for the purpose they are intended. Get into the habit of using the formulas directly, computing by the engineers' short method, by using logarithms or a slide rule. It is good practice and you know at all times what you are doing.

Do not assume that graphical and mechanical means of solution are not desirable.* They are, but only where you are going to solve similar problems over and over again. This is usual where one specializes in designing similar devices.

REARRANGING FORMULAS

Quite often we remember or find a formula which is not set up for ready solution of the unknown, that is we find the unknown factor on the right-hand side with the known factors. We may use the formula as given or rearrange it into the usual form; unknown factor on the left, known factors on the right. A simple example will bring out what is meant. Take the important basic formula:

$$= \frac{1}{2\pi\sqrt{LC}} \qquad (1)$$

Where f is in c. p. s.
L is in henries
C is in farads

Suppose we have a problem where we know the frequency involved and have a condenser which we wish to use. We want to know what inductance together with the available capacitor will give resonance at the frequency f. If formula (1) was arranged so L was on the left and C and f on the right, we could solve our problem by direct solution. How can we go about arranging the formula into this form, assuming that we do not know the new formula? For such a procedure you must have a suitable knowledge of algebra.*

Algebra tells us that if we perform the same operation to both sides of an equation we have not destroyed its validity as a correct equation. So, in the above case, let us square both sides of the formula. We get:

$$f^2 = \frac{1}{4\pi^2 LC} \qquad (2)$$

Now let us multiply both sides by L, which gives us:

$$f^2 L = \frac{L}{4\pi^2 LC} \qquad (3)$$

We may now cancel the L in the numerator and the L in the denominator of the right-hand term, which then gives:

$$f^2 L = \frac{1}{4\pi^2 C} \qquad (4)$$

Now let us divide both sides of equation (4) by f^2, and get:

$$\frac{f^2 L}{f^2} = \frac{1}{4\pi^2 C f^2} \qquad (5)$$

Cancelling f^2 on the left-hand side, we get the desired formula:

$$L = \frac{1}{4\pi^2 C f^2} \qquad (6)$$

Most beginners, when they rearrange a formula, are doubtful of its correctness.

A simple check of the algebraic manipulations is easily made. We know that the original formula (1) is correct. Assume values for the unknown, in fact any value Let us say that L is 2 henries, C is 2 farads. Of course, 2 farads is an absurd value, but it does not matter in a check. Substitute these values in formula (1) (the original), and we find that:

$$= \frac{1}{2\pi\sqrt{2 \times 2}} \quad \frac{1}{6.28\sqrt{4}} = \frac{1}{6.28 \times 2}$$

$$= \frac{1}{12.56} = .0796 \text{ c.p.s.}$$

Assume that the value of .08 is close enough for our present needs. Now if the derived formula (6) is correct, we should get a value of 2 for L, when we substitute

* This subject is beyond the scope of the average N. R. I. student. For a man with an advanced knowledge of mathematics we suggest Lipka's book, "Graphical and Mechanical Computation," published by John Wiley and Sons, Inc., N. Y. C. Price, $4. Considers alignment charts in detail.

* We suggest that you study such texts as
Mathematics for Electricians and Radio Men, by Nelson M. Cooke. Published by McGraw-Hill Book Co., Inc., N. Y. C.
Algebra for the Practical Man, by J. E. Thompson Published by D. Van Nostrand Co., Inc., N. Y. C.
Practical Mathematics, Part II, by C. I. Palmer Published by McGraw-Hill Book Co., Inc., N. Y. C.

.08 (closest value to .0798) for f and 2 for C. Let us try this. Thus by substitution:

$$L = \frac{1}{4\pi^2 C f^2}$$
$$= \frac{1}{4\pi^2 \times 2 \times .08^2}$$
$$= \frac{1}{4 \times 3.14 \times 3.14 \times 2 \times .08 \times .08}$$
$$= \frac{1}{.505} = \text{approx. } 2$$

we have proved that the derived formula is correct.

Quite often it is not easy to rearrange a formula in the standard form. In fact, when a problem arises where the unknown is on the right with known factors, experienced technicians don't even try to derive a suitable formula. They make immediate substitutions and solve by algebra for the unknown. Let us consider a simple example.

A radio amateur desires to build a transposed feeder line between the antenna and his transmitter. The line is to have a surge impedance of 440 ohms and he wants to use standard transportation blocks that place the two feeder wires 2 inches apart. There is a formula for the surge impedance involving the factors

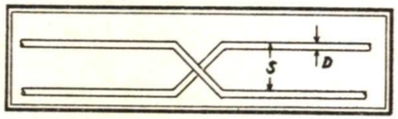

FIG. 14

given in Fig. 14. It is:

$$Z_o = 277 \log_{10} \frac{2S}{D}^* \qquad (7)$$

Where Z_o is the surge impedance in ohms
S and D are measured in the same dimensions, inches or centimeters.

From our problem we know that Z_o is 440 ohms and S is 2 inches. We want to know what the value of D should be. We may rearrange the formula, by algebra† or we may substitute the values and solve for S, as we shall in this case. Substituting the known values in formula (7), we get:

$$440 = 277 \log \frac{2 \times 2}{D} \qquad (8)$$

* There are two standard logarithms, base 10 and base ϵ (2.72+). It is customary practice to signify only the ϵ base by a subscript thus $\log_\epsilon 49$. Because the base 10 is so common the subscript $\log_{10}$ is omitted. In this text $\log x$ will mean to the base 10.

† By rearrangement we get:

$D = \frac{2S}{\log^{-1} Z_o/277}$ where $\log^{-1}$ means a number whose log is equal to the value of $Z_o/277$.

By arithmetic we reduce this to:

$$1.59 = \log 4/D \qquad (9)$$

What this equation says is that the logarithm of $4/D$ is equal to 1.59. Now what number would have the logarithm 1.59? From a log table we find that the number 38.9 would have that logarithm. Check this yourself. The characteristic of 38.9 is 1 and the mantissa is .5899, which is close enough to 0.59. Now we may say that:

$$4/D = 38.9 \qquad (10)$$

or

$$D = \frac{4}{38.9} = .103 \text{ inch} \qquad (11)$$

Referring to a wire table we find that a No. 10 B & S gauge wire would have a diameter of .102 inch. Therefore, the amateur would use this size wire.

COMBINING FORMULAS

We mentioned that radio experts who find formulas of particular use in their work, memorize certain basic formulas and derive the ones they need. Two cases have been cited. This sort of formula manipulation may be greatly extended, and to cases where formulas are introduced into one another. Take the formula:

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \qquad (1)$$

which you will recognize as the impedance formula for a resistance, inductive reactance and capacitive reactance in series. This formula is always memorized.

Now consider the same circuit without capacitive reactance, that is in formula (1), $X_c = 0$. This gives us at once the formula:

$$Z = \sqrt{R^2 + X_L^2} \qquad (2)$$

If the inductive reactance is zero, that is $X_L = 0$, we get:

$$Z = \sqrt{R^2 + X_c^2} \qquad (3)$$

In the latter case we must realize that $-X_c^2$ equals $+X_c^2$, as taught in a course in algebra.

Suppose we do not know the inductive or capacity reactance, but know the line frequency and the inductance and capacity. There are two basic formulas that tell us that:

$$X_L = 2\pi f L \qquad (4)$$
$$X_c = \frac{1}{2\pi f C} \qquad (5)$$

If we substitute formulas (4) and (5) in formula (1), we get:

$$Z = \sqrt{R^2 + \left(2\pi f L - \frac{1}{2\pi f C}\right)^2} \qquad (6)$$

which is a very important formula in A.C. circuit theory.

The ability to combine and interpret formulas is very valuable to the advanced radio technician. We will consider a very valuable case.

Suppose we have a resonant circuit consisting of a fixed coil shunted by a variable condenser. We know from experience that it will tune to a maximum and minimum frequency. Can we derive a formula that will tell us quickly what the ratio of the maximum to minimum frequency will be without tedious computation? We always start by setting down algebraically the statement that interests us, and then simplify. Suppose we consider the case where the inductance is L, the maximum capacity is C_1 and the minimum capacity is C_2. We must be sure that C_1 and C_2 include the distributed capacity of the coil.

We start with the basic formula:

$$= \frac{1}{2\pi \sqrt{LC}} \qquad (7)$$

For the maximum capacity of the condenser (100 dial position) we may say:

$$f_1 = \frac{1}{2\pi \sqrt{LC_1}} \qquad (8)$$

For the minimum capacity setting (0 dial setting) we may say:

$$f_2 = \frac{1}{2\pi \sqrt{LC_2}} \qquad (9)$$

We know that f_2 will be larger than f_1, so let us determine what the ratio of f_2 to f_1 is. Let us set this down algebraically thus:

$$\frac{f_2}{f_1} = \frac{\frac{1}{2\pi\sqrt{LC_2}}}{\frac{1}{2\pi\sqrt{LC_1}}} \qquad (10)$$

The next steps involve simplifying expression (10). Multiply the numerator and denominator of the right-hand side by 2π. This gives:

$$\frac{f_2}{f_1} = \frac{\frac{1}{\sqrt{LC_2}}}{\frac{1}{\sqrt{LC_1}}} \qquad (11)$$

Now multiply both numerator and denominator by $\sqrt{LC_2}$ and $\sqrt{LC_1}$. This simplifies the expression to:

$$\frac{f_2}{f_1} = \frac{\sqrt{LC_1}}{\sqrt{LC_2}} \qquad (12)$$

and finally:

$$\frac{f_2}{f_1} = \sqrt{\frac{C_1}{C_2}} \qquad (13)$$

With this formula it is simpler to tell through what range a given tuned circuit will respond. For example an R.F. broadcast tuned circuit may have a capacity variation of 9 to 1. In which case

$$\frac{f_2}{f_1} = \sqrt{\frac{9}{1}} = 3 \qquad (14)$$

This tells us that the f_2 will be 3 times f_1.

In a manner similar to the way formula (13) was derived, we can obtain the more complete formula where the inductance and capacity may vary. This is given by:

$$\frac{f_2}{f_1} = \sqrt{\frac{L_1 C_1}{L_2 C_2}} \qquad (15)$$

A PRACTICAL PROBLEM IN DESIGN

As a practical problem, let us consider the design of an oscillator and pretuner tuning stages of a superheterodyne so that they will track. We will assume, if two tie-down points are realized, that satisfactory tracking may be arranged by trimmer adjustments. The two tie-down points we will assume are 1400 and 600 kc. From the theory and practice of padding we know that, at 1400 kc., enough turns are taken off the oscillator coil so that a tie-down is obtained and so that the oscillator frequency is above the signal frequency by the I.F. value. At the lower frequency (600 kc.) the number of turns taken off are insufficient, so a so-called padding condenser is placed in series with the tuning condenser in the oscillator circuit so a second tie-down is obtained. The insertion of the padding condenser has little effect at the high frequency, and whatever upset is obtained may be corrected by a trimmer. Obviously the important problems in this design are the oscillator coil inductance and the value of the padding condenser.

Assume that the inductance of the coils in the preselector is 250 μh and the I.F. is 175 kc. From formula (13):

$$\frac{f_{p1}}{f_{o1}} = \sqrt{\frac{L_o}{L_p}}^* \qquad (16)$$

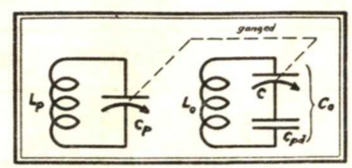

FIG. 15

we may now determine the value of the

* The notations are: p for pretuner, o for oscillator, 1 for 1400 kc., and 2 for 600 kc. Thus f_{p1} is the resonant frequency of the pretuner at 1400 kc.

oscillator inductance L_o for this tie-down point.

Substituting in the formula we get:

$$\frac{1400}{1575} = \sqrt{\frac{L_o}{250}} = .89 \qquad (17)$$

Squaring the above equation, we get:

$$\frac{L_o}{250} = .792 \qquad (18)$$

Therefore $L_o = .792 \times 250 = 198\ \mu h.$

Now let us turn our attention to getting a suitable tie-down at 600 kc., knowing that $L_p = 250\ \mu h$ and $L_o = 198\ \mu h$. From formula:

$$\frac{f_{p2}}{f_{o2}} = \sqrt{\frac{L_o\, C_{o2}}{L_p\, C_{p2}}} \qquad (19)$$

we may determine the value of C_{o2}, that is the net value of C_{pd} the padding condenser in series with C for that position (see Fig. 15). We need to know the value of C_{p2}. First let us compute C_{p2} from the formula:

$$C_{p2} = \frac{25330}{f^2_{p2}\, L_p}\ \mu f. \qquad (20)$$

Where f_{p2} is in kc.
L_p is in μh.

Substituting into the expression we get:

$$C_{p2} = \frac{25330}{600 \times 600 \times 250} \qquad (21)$$

$$= \frac{25330}{90{,}000{,}000}$$

$$= .000282\ \mu f.$$

$$= 282\ \mu\mu f.$$

Now we may substitute into expression (19). If we express C_{p2} in micro-micro-farads, we obtain C_{o2} in the same units. Substituting we get:

$$\frac{600}{775} = \sqrt{\frac{198 \times C_{o2}}{250 \times 282}} \qquad (22)$$

simplifying:

$$.774 = \sqrt{\frac{C_{o2}}{356}} \qquad (23$$

Squaring both sides gives:

$$.600 = \frac{C_{o2}}{356} \qquad (24)$$

and:

$$C_{o2} = 214\mu\mu f. \qquad (25)$$

Obviously while the condenser C is set to have a capacity of 282 $\mu\mu f.$, the net oscillator coil shunting capacity should be 214 $\mu\mu f.$ As stated, the padding condenser is used for this purpose. For the two condensers in series the net capacity is determined from the formula:

$$C_{o2} = \frac{CC_{pd}}{C + C_{pd}} \qquad (26)$$

Substituting we get:

$$214 = \frac{282 \times C_{pd}}{282 + C_{pd}} \qquad (27)$$

Multiplying both sides by $(282 + C_{pd})$ we obtain:

$$60400 + 214C_{pd} = 282C_{pd} \qquad (28)$$

$$60400 = 68C_{pd} \qquad (29)$$

and $$C_{pd} = \frac{60400}{68} = 890\mu\mu f. \qquad (30)$$

In actual practice the padding condenser may be 850 $\mu\mu f.$ shunted by a 100 $\mu\mu f.$ trimmer. If the system is designed with the calculated values and the pretuner and oscillator aligned in the usual manner, very little trouble will be experienced.

CONCLUSIONS

In this short lesson on formulas and their use, we have shown how valuable a formula may be for explaining theory, how they may be used in design, and how they may help in servicing. We merely wish to add that if you have mastered your radio theory, and can select the appropriate formula, and learn how to juggle and compute with formulas, you can make formulas do "tricks" for you.

RADIO FORMULAS

A: FUNDAMENTAL RADIO—ELECTRIC CIRCUIT LAWS

Governing the entire theory of radio circuits are certain extremely important basic laws. With these laws, advanced radio engineers and scientists have developed many of the formulas given in this text. Experience has shown that a number of special problems are solved quicker by starting with the fundamental circuit laws. Many of these laws are valuable in visualizing what goes on in the circuit. Without regard to their relative importance, these laws are as follows:

Kirchoff's Laws

Law 1. The sum of all the currents flowing towards a junction (connection) in any network of conductors is equal to all the current flowing away from the junction. This

law may also be stated as: The algebraic sum of all the currents toward a junction is zero. By the algebraic sum we mean that if the current toward the junction be considered + or positive; the current away from the junction shall be considered — or negative. Alternatively we may assume current "away" as +, and current "to" as —. As a formula, the law may be written:

$$\Sigma I = 0 \qquad (1A)$$

where the symbol Σ is read as "sum of." It is the Greek letter "sigma."

Law 2. In any complete circuit of any network the sum of all the voltages generated (e.m.f.'s) are equal to the algebraic sum of all the voltage drops (impedance or resistance drops). If we consider all the e.m.f.'s as voltage rises, we may state this law as: The sum of the voltage rises plus all the voltage drops in a complete circuit is equal to zero. Expressed as a formula, this is written as:

$$\Sigma E = 0 \qquad (2A)$$

Example of Kirchoff's Laws:

Given the supply and load circuit shown in Fig. 1A

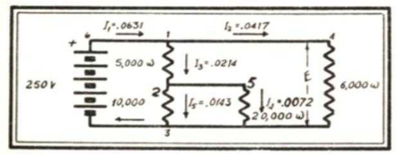

FIG. 1A

Observe that there are three junction points, namely: 1, 2, and 3. In all cases Law 1 ($\Sigma I = 0$) is true.

For
(1) .0631 − .0417 − .0214 = 0
(2) .0214 − .0072 − .0143 = 0 (close enough)
(3) .0072 + .0143 + .0417 − .0631 = 0

The only voltage rise in this circuit is the 250 volts, and it may be a battery, generator or the equivalent of the output of a rectifier. All other voltages of the circuit are considered voltage drops. In this case each voltage drop is equal to a resistance times the current flowing through the resistor. We have in this network three complete circuits in which e.m.f.'s and voltage drops are concerned. There are also other complete circuits if they are of value in finding a solution. Let us take the circuits each with an e.m.f. Considering the fact that $\Sigma E = 0$.

Circuit 6-1-2-3-6 according to this law gives:

250 − .0214 × 5000 − .0143 × 10,000 =
250 − 107 − 143 = 0

Circuit 6-1-4-3-6 gives:

250 − .0417 × 6000 = ?
250 − 250 = 0

Circuit 6-1-2-5-3-6 gives:

250 − .0214 × 5000 − .0072 × 20,000 = ?
250 − 107 − 144 = 0 (close enough)

Note that in this example we have given all the details of the circuit and proved by simple computations that Kirchoff's laws are true. Accordingly we have shown that the values are correct. This is of particular value to the practical technician, where circuit values are given and he wants to prove that they are correct. For the designer there is a greater use for Kirchoff's laws. Given certain facts about the circuit, he may want to find the remaining facts. For example, suppose the generated voltage and the resistance were known. He wants to know the currents. Using laws 1 and 2, he would set up as many equations as there were unknown currents to be determined and solve these simultaneous equations as they are called, by algebra. The equations for this circuit would be:

(1) $250 - 6000I_2 = 0$ (circuit 6–1–4–3–6)

(2) $250 - 5000I_3 - 10{,}000I_5 = 0$ (circuit 6–1–2–3–6)

(3) $250 - 5000I_3 - 20{,}000I_4 = 0$ (circuit 6–1–2–5–3–6)

(4) $I_1 - I_2 - I_3 = 0$ (Junction 1)

(5) $I_3 - I_4 - I_5 = 0$ (Junction 2)

For the solution of these equations we refer you to a text on algebra.

Ohm's Law.

Ohm's law may be stated in a number of ways. The most common statement is:

(a) The current through a resistance or a reactance is the voltage applied divided by the resistance or reactance. Stated as formulas we have:

$$I = E/R \qquad (3A)$$

$$I = E/X \qquad (4A)$$

Where I is in amperes
E is in volts
R and X are in ohms

We must recognize the fact that X may be inductive or capacitive reactance and that the inductive reactance X_L is equal to $2\pi fL$, while the capacitive reactance X_c is equal to $1/2\pi fC$.

Of course we may have a device that has resistance and reactance, the net being referred to as impedance Z. Ohm's law must then be written as:

$$I = E/Z \qquad (5A)$$

In the general case of a device having resistance, inductance and capacitance in series, Z in this formula is equal to:

$$\sqrt{R^2 + (2\pi fL - 1/2\pi fC)^2}$$

(b) Ohm's law is also stated as follows: The current through an inactive or passive device (a device which does not itself generate a voltage) is proportional to the voltage applied. Stated as a formula, we have:

$$I = GE \qquad (6A)$$

Where I is in amperes
E is in volts
G is in mhos

In this case G is referred to as the conductance.

For an inductance or capacitance this law is algebraicly expressed as:

$$I = BE \qquad (7A)$$

Where B is in mhos

The symbol B is referred to as the susceptance of the device and is the reciprocal of reactance, that is:

$$B = 1/X \qquad (8A)$$

Thus, for an inductance B_L is equal to $1/2\pi fL$ and for a capacitance B_c is equal to $2\pi fC$. Where the passive device or network includes susceptance and conductance, the sum effect is called the admittance, Y. Ohm's law therefore becomes.

$$I = YE \qquad (9A)$$

Where Y is in mhos

In the general case of a circuit having resistance, inductance and capacitance in parallel, Y in this formula is equal to:

$$\sqrt{G^2 + (2\pi fC - 1/2\pi fL)^2}$$

The importance of using resistance, reactance and impedance in one case and using conductance, susceptance and admittance in the other case arises from the fact that in series circuits, R, X, and Z may be added to get the resultant, while in parallel circuits G, B, and Y may be added to get the resultant.*

The Principle of Superposition.

In any network consisting of resistances, inductances and capacitances which do not change in value, the currents produced by the presence of many varied voltages (e.m.f.'s) may be considered to be the sum of the currents produced by the individual e.m.f.'s.

For example, if the voltage consists of a fundamental, a third and a fifth harmonic, the currents flowing may be considered first for the fundamental, then for the third harmonic and finally for the fifth harmonic. The total current at any point in the circuit then is the sum of the three. The absolute value of the current will be given by the formula:

$$I = \sqrt{I_1^2 + I_2^2 + I_3^2 + \text{etc.}} \qquad (10A)$$

If I_1, I_2, etc. is given in root mean square value, I will be in r.m.s.

In a number of circuits where the resistance, capacitance and inductance do vary, it is usual for initial purposes to assume that they are constant. Corrections or limitations are then necessary to qualify the actual and apparent conditions.

The Reciprocity Theorem.

If any type of e.m.f. located at one point in a circuit network produces a current at any other point in the network, then the same e.m.f. located at the second point would produce the same current at the first point.

This theorem does not apply to vacuum tubes, rectifiers or devices where the circuit acts only in one direction. The theorem is helpful in filter, transmission line and general circuit design. If E is the voltage acting at point 1, and I the current produced at the second point, then E/I is referred to as the transfer impedance. In short it reduces a complex device or network to a simple impedance.

Thevenin's or Pollard's Theorem.

A very important principle which states: If an impedance Z is connected between any two points in a network, the resultant current I through the added impedance will be given by dividing the voltage E existing across the two points prior to connecting the impedance, divided by Z plus the impedance Z_1 that would be measured across the two points prior to the connection of the impedance. In calculating the impedance Z_1 the e.m.f.'s are considered inactive. Thevenin's theorem in equation form is:

$$I = \frac{E}{Z + Z_1} \qquad (11A)$$

Obviously the new voltage across the two points will be IZ, and thus the new terminal conditions are determined.

This theorem is quite valuable when some load is to be added to an existing circuit and the new terminal conditions are to be determined quickly.

* It should be remembered that X, Z, B, and Y, must be considered as vectors and so treated when adding. We refer you to any standard text on the fundamentals of electrical engineering.

Compensation Theorem

An impedance in any circuit may be replaced by a generator (with no internal impedance) which at every instant duplicates the voltage that appears across the replaced impedance.

This principle is extremely useful in representing such devices as microphones and vacuum tubes or networks as equivalent generators.

For purposes of substitution the theorem as it is now to be stated has a more practical value. If a network is modified by changing one portion of it by a *change in impedance,* the effect in any other portion of the circuit would be the same as if the change were made by an e.m.f. acting in series with the modified impedance and equal to the *change in impedance* times *the current* through that impedance before the change was made.

Points of Equal Potential

It is convenient at times when considering complicated networks to consider points of equal potential as electrically connected by a wire of zero impedance. An example of this is the balanced wheatstone bridge.

Short Circuit Current Solution of Circuits

All the principles outlined so far are used in solving circuit problems. Quite often the process is lengthy and tedious. The short circuit current solution given now is at times a superior method. This method is particularly suitable in solving circuits where several generators feed a load or a passive network. The principle is stated as follows:

The voltage across the real load is equal to an equivalent load considering the load and the generator impedances in parallel multiplied by the sum of the short circuited currents of each generator, derived by considering the terminals of each generator (including its series impedance) shorted.

Example: Consider the simple circuit shown in Fig. 2A.

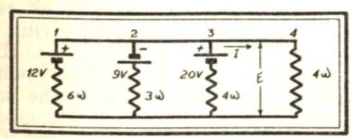

Fig. 2A

The equivalent resistance (we are dealing with pure resistances) is the sum of 6, 3, 4, and 4 ohms in parallel and equals:

$$\frac{1}{R} = \frac{1}{6} + \frac{1}{3} + \frac{1}{4} + \frac{1}{4} = \frac{2 + 4 + 3 + 3}{12}$$

Solving this by algebra, we get:

$$R = 1 \text{ ohm}$$

The short circuit currents of branches No. 1, No. 2, and No. 3 are:

$$I_{s1} = \frac{+12}{6} = +2$$

$$I_{s2} = \frac{-9}{3} = -3$$

$$I_{s3} = \frac{+20}{4} = +5$$

The total short circuit current:

$$I_S = +2 - 3 + 5 = +4$$

Therefore according to the short circuit theory:

$$E = 4 \times 1 = 4 \text{ volts}$$

Current in the load:

$$I = 4/4 = 1 \text{ ampere}$$

With the information given so far we may compute the currents in each branch, knowing that the terminal voltage of each generator is 4 volts. For example, in branch No. 2.

$$I_2 = \frac{-9 - 4}{3} = -4\tfrac{1}{3} \text{ amp.}$$

(Current flowing down)

ADVANCED CONCEPTS

Note that examples were taken where resistors were the only circuit elements. The same principles hold true if impedances, Z were used. Also observe that in some of the theorems the impedance factor was used. The same principle will hold if resistances R are used.

In solving a problem where impedances are found, we should consider the impedance as made up of two components referred to as the real and imaginary components. To distinguish the imaginary from the real, it is prefixed by the letter j. Thus an impedance is always written:

$$Z = R + jX$$

Whereas an admittance is always written:

$$Y = G + jB$$

The absolute value of Z, always written $|Z|$, is given by the formula:

$$|Z| = \sqrt{R^2 + X^2}$$

Whereas the absolute value of Y is written:

$$|Y| = \sqrt{G^2 + B^2}$$

The manipulation of such values, called *vector* quantities require a knowledge of electrical engineering and advanced algebra. This is beyond the scope of this course. Students with a suitable training will find the subject treated in standard Electrical Engineering texts* and texts on Algebra.† Students interested in advanced radio engineering may consider this a subject for advanced study. In the following formulas only the absolute values, as read by a voltmeter or ammeter are to be considered.

* Communication Engineering by Everitt, published by McGraw-Hill Book Company, Chapters II and III.

† Algebra for the Practical Man by Thompson, published by D. Van Nostrand Co., Chapter VIII.

B: RESISTORS

Resistance from Dimensions

$$R = \rho L/A \qquad (1B)$$

Where R is in ohms
L length
A the cross section area
ρ the resistance per unit length and cross section.
If L is in feet, A in circular mils; ρ is the resistance in ohms for a wire one foot long and having a cross section of one circular mil. See special electrical tables: $\rho_{copper} = 10.4$; $\rho_{alum} = 17.1$; $\rho_{nichrome} = 600$; etc.

Conductance from Dimensions

$$G = \gamma A/L \qquad (2B)$$

Where A is the cross section area
L is the length
G is in mhos
γ is the conductivity

$$\gamma = 1/\rho \qquad (3B)$$

Resistance at a New Temperature (°C)

$$R_t = R_o\,(1 + \alpha t) \qquad (4B)$$

Where t is the new temperature degrees Centigrade
R_o is the resistance at 0° C
α is the temperature coefficient
R_t is the resistance (ohms at $t°$ C)

Formula (4B) may be more conveniently used as:

$$R_{t2} = R_{t1}\,[1 + \alpha(t_2 - t_1)] \qquad (5B)$$

Where t_2 is the final temp.
t_1 is the initial temp.
R_{t2} the resistance at t_2
R_{t1} the resistance at t_1
α the temperature coefficient, for example: $\alpha_{copper} = .00393$; $\alpha_{alum} = .0039$; etc.

Temperature Rise in Electrical Conductors

$$\Delta t = \frac{1}{\alpha}\left(\frac{R_{t2}}{R_{t1}} - 1\right) \qquad (6B)$$

Where Δt is the temperature rise

As copper wire is extensively used, the practical formula becomes:

$$\Delta t = 254\left(\frac{R_{t2}}{R_{t1}} - 1\right) \qquad (7B)$$

Add Δt to t_1, the original temperature of the surroundings, to find temperature of the conductor.

Resistors in Series

$$R = R_1 + R_2 + R_3 + R_4 + \text{etc.} \qquad (8B)$$

Where R, R_1, etc., are in ohms
R_1, R_2, R_3, etc., are the series elements
R is the total resistance

When $R_1 = R_2 = R_3$, etc.:

$$R = nR_1 \qquad (9B)$$

Where R_1 is the resistance of one resistor
n is the number of resistors

Resistors in Parallel

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} + \text{, etc.} \qquad (10B)$$

For three resistors in parallel:

$$R = \frac{R_1R_2R_3}{R_2R_3 + R_1R_3 + R_1R_2} \qquad (11B)$$

For two resistors in parallel:

$$R = \frac{R_1R_2}{R_1 + R_2} \qquad (12B)$$

For n resistors of equal value R_1 in parallel:

$$R = R_1/n \qquad (13B)$$

Conductance

$$G = \frac{1}{R} \qquad (14B)$$

Where R is in ohms
G is in mhos

Conductors in Parallel

$$G = G_1 + G_2 + G_3 + G_4\text{, etc.} \qquad (15B)$$

Where G_1, G_2, etc., are the conductances of the devices

Equivalent Delta (π) of Star (T) or Vice-Versa

In reducing complex circuits to simple circuits, it is convenient in some cases to convert a delta (called in radio a π) circuit into a star (called in radio a T) circuit. The reverse may be the case. See Fig. 1B for notation. If point A is grounded, point B with the ground is considered the input, while point C with the ground is considered the output, then the familiar π and T circuits in radio will be recognized.

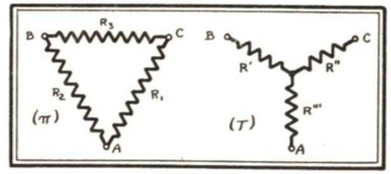

Fig. 1B

*To change a π to a T**

$$R' = \frac{R_2R_3}{R_1 + R_2 + R_3} \qquad (16B\text{–}1)$$

$$R'' = \frac{R_1R_3}{R_1 + R_2 + R_3} \qquad (16B\text{–}2)$$

$$R''' = \frac{R_1R_2}{R_1 + R_2 + R_3} \qquad (16B\text{–}3)$$

*To change a T to a π**

$$R_1 = \frac{R'R'' + R''R''' + R'R'''}{R'} \qquad (17B\text{–}1)$$

$$R_2 = \frac{R'R'' + R''R''' + R'R'''}{R''} \qquad (17B\text{–}2)$$

$$R_3 = \frac{R'R'' + R''R''' + R'R'''}{R'''} \qquad (17B\text{–}3)$$

Power Loss

$$P = I^2R \qquad (18B)$$

Where P is in watts
I is in amperes
R is in ohms

$$P = E^2/R \qquad (19B)$$

Where E is the voltage drop across R

$$P = EI \qquad (20B)$$

Where E is the voltage across
I the current through the load

* We may substitute Z for R if we deal with impedances.

C: CONDENSERS (STATIC)

Capacity from Dimensions

Two Plates

$$C = .225\,\frac{KA}{d} \qquad (1C)$$

Where C is in micro-microfarads, $\mu\mu f$
A is area in square inches of one plate meshing with the other
d is the plate separation in inches
K is the dielectric constant of the separating medium. $K_{air} = 1.0$; $K_{glass} = 8$ to 9; $K_{castor\ oil} = 13.0$

Several plates

$$C = .225\,\frac{KA}{d}\,(n - 1) \qquad (2C)$$

Where n is the number of plates
A is in square inches
d is in inches

$$C = .0885\,\frac{KA}{d}\,(n - 1) \qquad (3C)$$

Where A is in square centimeters
d is in centimeters

Plates to Remove for Desired Capacity

$$N_o = (N_1 - 1)\,\frac{C_o}{C_1} + 1 \qquad (4C)$$

Where N_o is the remaining number of plates
N_1 is the original number of plates
C_o is the desired capacity
C_1 is the original capacity

Capacity of Two Parallel Wires

$$C = \frac{3.68}{\log\left(\frac{2D}{d}\right)} \qquad (5C)$$

Where C is in micro-microfarads per foot
D is the separation of wires (center to center)
d is the diameter of the wire
d and D must be in the same units

Capacity of Round Wire Surrounded by a Round Tube

$$C = \frac{7.35K}{\log\left(\frac{r_o}{r_i}\right)} \qquad (6C)$$

Where C is micro-microfarads per foot
K is the dielectric constant of separating medium
$K_{air} = 1$. Assume $K = 1$ where beads are used only infrequently for spacers
r_i is the radius of the inner wire
r_o is the radius of the inner surface of the outer conductor
r_i and r_o are in the same units

Condensers in Parallel

General

$$C = C_1 + C_2 + C_3 + C_4 + \text{etc.} \qquad (7C)$$

Where C_1, C_2, C_3, etc. are in the same units, and represent the respective capacities of the condensers
C is the total capacity, same units as C_1, etc

Equal Condensers in Parallel

$$C = nC_1 \qquad (8C)$$

Where C_1 is the capacity of one condenser
n is the number in parallel

Condensers in Series

General

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4} + \text{etc.} \qquad (9C)$$

Where C_1, C_2, etc. are the respective capacities of the condensers in series
C is the total capacity

Equal Condensers in Series

$$C = C_1/n \qquad (10C)$$

Where n is the number in series

Three Condensers in Series

$$C = \frac{C_1C_2C_3}{C_2C_3 + C_1C_3 + C_1C_2} \qquad (11C)$$

Where C_1, C_2, and C_3 are the capacities of the three condensers

Two Condensers in Series

$$C = \frac{C_1C_2}{C_1 + C_2} \qquad (12C)$$

Charge in a Condenser

$$Q = CE \qquad (13C)$$

Where C is in farads
E is in volts
Q is in coulombs

Energy Stored in a Condenser

$$W = 0.5\ CE^2 \qquad (14C)$$

Where C is in farads
E is in volts
W is in joules

Elastance of a Condenser

$$S = 1/C \qquad (15C)$$

Where C is in farads
S is in darafs

D: COILS (STATIC)

Inductance of Coils in Series.

$$L = L_1 + L_2 + L_3 + L_4 + \text{etc.} \qquad (1D)$$

Where L, L_1, etc. are in the same units; henries, millihenries, microhenries. No coupling between coils.

Inductance of Coils in Parallel (No Coupling)

$$\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \text{etc.} \qquad (2D)$$

Inductance of Two Coils with Coupling

$$L = L_1 + L_2 \pm 2M \qquad (3D)$$

+ for aiding
− for opposing

Inductance of Single Layer Solenoids*

$$L = FdN^2 \qquad (4D\text{–}1)$$

Where L is in microhenries
N is the number of turns
d is the coil diameter in cms or inches
F the factor determined from curve Fig. 12, page 11

$$L = \frac{0.41\ a^2N^2}{9a + 10b} \qquad (4D\text{–}2)$$

Where a and b are as indicated in Fig 1D, and are in Centimeters, Multiply inches by 2.54 to get Centimeters.

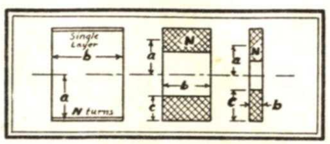

Fig. 1D

* All inductance formulas are approximations and in radio work should be checked by measurements and turn adjustments made.

Inductance of Multilayer Round Coils

See Fig. 1D. Applies to honeycomb, smooth layer and jumbled layer windings. When the width b is greater than the radius a:

$$L = \frac{.314a^2\ N^2}{6a + 9b + 10c} \qquad (5D)$$

Where L is in microhenries
a, b, c dimensions are in centimeters
When dimensions are in inches, multiply answer by 2.54 to get L in μh.

For a pancake coil where b is much less than c (applies strictly to a coil in which the b dimension is one layer wide):

$$L = \frac{.41a^2\ N^2}{8a + 11c} \qquad (6D)$$

Where L is in microhenries
dimensions in centimeters

For a square cross section coil $b = c$ and $a = 3/2\ c$ (diameter of core equals $2c$):

$$L = .064\ CN^2 \qquad (7D)$$

Where L is in μh
C is width and height in inches
N is the number of turns

Mutual Inductance of Coaxial Solenoids

The following formula is only approxi-

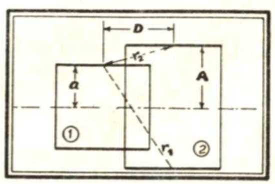

Fig. 2D

mate. Follow the given procedure. First find the value of r_1 and r_2 by the formulas:

$$r_2 = \sqrt{\left(1 - \frac{a}{A}\right)^2 + \frac{D^2}{A^2}} \qquad (8D\text{–}1)$$

$$r_1 = \sqrt{\left(1 + \frac{a}{A}\right)^2 + \frac{D^2}{A^2}} \qquad (8D\text{–}2)$$

Then find the ratio of $\frac{r_2}{r_1} = K$

Find N from the following graph, corresponding to K:

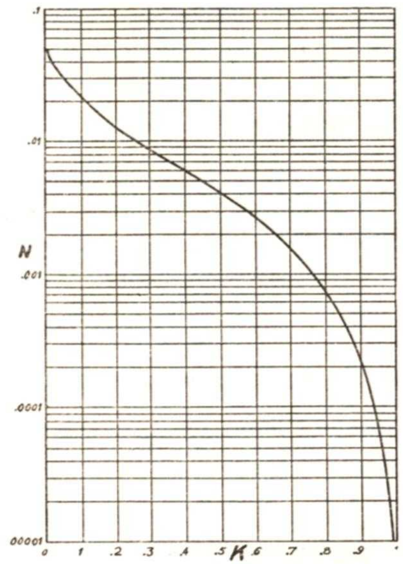

Fig. 3D

Compute the value of M_o from the formula:

$$M_o = N\ \sqrt{A \times a} \qquad (8D\text{–}3)$$

Where N is obtained from Fig. 3D
A and a from Fig. 2D (in cms.)

And finally by substituting M_o in:

$$M = n_1\ n_2\ M_o \qquad (8D\text{–}4)$$

Where M is in microhenries
n_1 is the turns in coil 1
n_2 is the turns in coil 2

The process holds true for single layer coils whose length is equal to the diameter. The method is close enough for other coils. r_1 and r_2 are measured to the centers of the wound layers.

Distributed Capacity Single Layer Coils

$$C_o = .6r \qquad (9D\text{–}1)$$

Where C_o is in $\mu\mu f$.
r is the coil radius in cms.

$$C_o = .76D \qquad (9D\text{–}2)$$

Where C_o is in $\mu\mu f$.
D is the coil diameter in inches

Chokes

C_o and the coil inductance L form an anti-resonant circuit at $\omega = 1/\sqrt{LC_o}$, where $\omega = 2\pi f_o$. Choke is inductive 0 to f_o, $2f_o$ to $3f_o$, $4f_o$ to $5f_o$, $6f_o$ to $7f_o$, etc. Choke is capacitive f_o to $2f_o$, $3f_o$ to $4f_o$, $5f_o$ to $6f_o$, etc. Maximum impedance at f_o, $3f_o$, $5f_o$, etc., and zero impedance at $2f_o$, $4f_o$, $6f_o$, etc.

Inductance of Two Parallel Wires

$$L = .281 \log \frac{2D}{d} + .030 \qquad (10D)$$

Where L is μh per foot of the transmission line formed by the 2 wires
D is center to center separation of wires
d is the diameter of the wire

Inductance of a Round Wire Surrounded By a Round Tube

$$L = .140 \log \frac{r_o}{r} + .015 \qquad (11D)$$

Where L is μh per foot of transmission line formed
r_o is radius of inner surface of outer tube
r_1 is radius of inner wire

Suggestions in Coil Design

When dimensions of a coil are given, the inductance calculation requires simple substitutions in the proper formula. When a definite inductance or mutual inductance is desired and the dimensions are to be found, the following procedure may be used. Find the proper formula. If certain dimensions are fixed by practical needs, they should be substituted into the formula. Assume various values for the other dimensions and calculate the inductance. Remember that inductance roughly increases as the square of the turns, square of the diameter and inversely as the length. By these facts you may approximate a better value, and thus approach values that give the desired L.

To find the length of a coil assume a wire size and insulation covering and from a wire table find the number of turns per inch. For a single layer coil the number of turns divided by the turns per inch give the coil length. For a multilayer coil this should be divided by the number of layers.

Coils with Iron Cores

$$L = 1.26\, N^2 P \times 10^{-8} \qquad (12D)$$

Where L is in henries
N is the turns
P is the permeance of the iron circuit, and is defined by the expression $\mu \frac{A}{l}$; μ being the permeability, A the cross-section, and l the length of the core, all dimensions in centimeters 10^{-8} = .00000001

For radio and audio frequency chokes with a polarizing D.C. current, P should be the A.C. permeance, for the frequency employed.

Energy Stored in a Coil

$$W = 0.5 L I^2 \qquad (13D)$$

Where W is in joules
L is in henries
I is in amperes

E: CIRCUITS HAVING ONLY RESISTANCES*

Ohm's Law

$$E = IR \qquad (1E\text{–}1)$$
$$I = E/R \qquad (1E\text{–}2)$$
$$R = E/I \qquad (1E\text{–}3)$$

Where E is in volts
I is in amperes
R is in ohms

Generator with Resistance

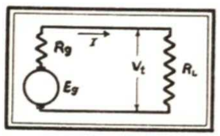

Fig. 1E

$$V_t = E_g - I R_g \qquad (2E\text{–}1)$$
$$E_g = I\,(R_g + R_L) \qquad (2E\text{–}2)$$
$$I = \frac{E_g}{R_g + R_L} \qquad (2E\text{–}3)$$

Where E_g is the no load or generated voltage
V_t the terminal voltage
I the line current
R_g the generator resistance in ohms
R_L the load resistance in ohms

E_g may be a generator; a battery or a vacuum tube, microphone or similar device which may by the compensation theorem be assumed as a generator.

Power Generated

$$P = E_g I \qquad (3E)$$

Where P is in watts
E is in volts
I is in amperes

* The formulas in this section apply to any A.C. or D.C. circuit with a resistance load. For A.C. circuits the r.m.s. values are considered.

Power Delivered

$$P = V_t I$$
$$P = I E_g - I^2 R_g \qquad (4E)$$

Efficiency of Circuit

$$Eff. = V_t / E_g \qquad (5E\text{–}1)$$
$$= \frac{R_L}{R_L + R_g} \qquad (5E\text{–}2)$$

To find *Eff.* in percent multiply by 100.

Maximum Power

Obtained when:

$$R_g = R_L \qquad (6E)$$

Voltage Generated by a D.C. Motor with Shunt Field

$$E_g = K\, I_f\, S \qquad (7E)$$

Where E_g is the average generated voltage
I_f the field current
S the speed in revolutions per minute
K is a constant for a given motor determined by setting I_f, S and measuring E_g, the no load voltage. $K = E_g / I_f S$. Formula holds good for values of I_f which do not produce magnetic core saturation.

Power Loss in a Resistance

$$P = I^2 R \qquad (8E\text{–}1)$$
$$P = VI \qquad (8E\text{–}2)$$
$$P = V^2/R \qquad (8E\text{–}3)$$

Where P is in watts
R is the resistor value in ohms
V is the voltage across the resistor
I is the current through the resistor

F: A.C. CIRCUITS WITH IMPEDANCE

Fundamental Concepts

Sine Waves

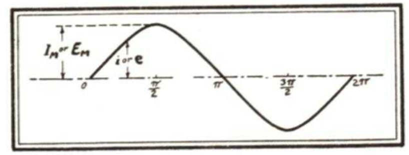

Fig. 1F

$$= I_M \sin \omega t \qquad (1F)$$
$$e = E_M \sin \omega t \qquad (2F)$$

Where i and e are the instantaneous values
I_M and E_M are the maximum or peak values
t is time in seconds
ω is the angular velocity and equals

$$\omega = 2\pi f \qquad (3F)$$

Where f is the frequency in cycles per second

Average Value

$$I_{AV} = .636\, I_M \qquad (4F\text{–}1)$$
$$E_{AV} = .636\, E_M \qquad (4F\text{–}2)$$

Root Mean Square Value

$$I_{R.M.S.} = .707 I_M \qquad (5F\text{–}1)$$
$$E_{R.M.S.} = .707 E_M \qquad (5F\text{–}2)$$

Phase Angle

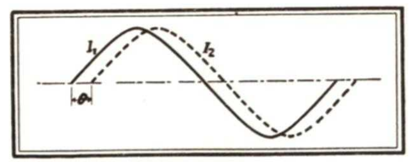

Fig. 2F

I_1 is the reference wave

I_2 leads I_1 by the angle θ.

Formulas indicating leading or lagging phase angles follow:

$$i_2 = I_2 \sin (\omega t + \theta) \text{ for leading wave} \qquad (6F\text{–}1)$$
$$i_2 = I_2 \sin (\omega t - \theta) \text{ for lagging wave} \qquad (6F\text{–}2)$$

To Change Degrees to Radian Angles

θ is usually expressed in degrees. When substituting in the sine formula it must be in radian angles.

$$\theta_r = \frac{\pi\, \theta_d}{180} \qquad (7F)$$

Where θ_r is the radian angles
θ_d is the angle in degrees

Representing Lagging and Leading Components

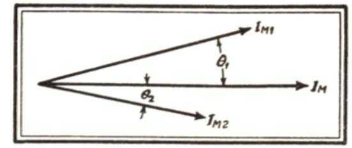

Fig. 3F

$$= I_M \sin \omega t \qquad (8F\text{–}1)$$
$$i_1 = I_{M1} \sin (\omega t + \theta_1) \qquad (8F\text{–}2)$$
$$i_2 = I_{M2} \sin (\omega t - \theta_2) \qquad (8F\text{–}3)$$

Power Factor

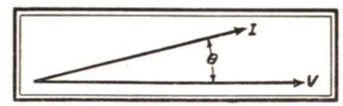

Fig. 4F

$$p.f. = \cos \theta \qquad (9F)$$

Power

$$P = VI \cos \theta \qquad (10F)$$
$$= VI \times p.f.$$

Where P is in watts
V voltage across device (rms value)
I current through device (rms value)

Reactance

$$X_L = 2\pi f L \qquad (11F\text{–}1)$$

Where X_L is reactance of coil in ohms
L is the inductance of coil in henries
f is the frequency of the current

$$X_c = 1/2\pi f C \qquad (11F\text{–}2)$$

Where C is the capacity of condenser in farads
X_c is the reactance in ohms

Condenser, Coil and Resistor in Series

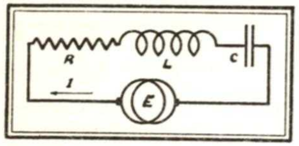

FIG. 5F

R, L and C in Series

$$Z = \sqrt{R^2 + (X_L - X_c)^2} \qquad (12F\text{–}1)$$

$$= \sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}$$

R and L in Series

$$Z = \sqrt{R^2 + X_L^2} \qquad 12F\text{–}2)$$

$$= \sqrt{R^2 + (2\pi fL)^2}$$

R and C in Series

$$Z = \sqrt{R^2 + X_c^2} \qquad (12F\text{–}3)$$

$$= \sqrt{R^2 + \left(\frac{1}{2\pi fC}\right)^2}$$

L and C in Series

$$Z = X_L - X_C \qquad (12F\text{–}4)$$

$$= 2\pi fL - \frac{1}{2\pi fC}$$

Current in Series Circuit

$$I = E/Z \qquad (13F\text{–}1)$$

Where Z is determined from (12F-1) to (12F-4)

Power Factor

$$p.f. = \frac{R}{Z} \qquad (13F\text{–}2)$$

Maximum Current I_o in Series Circuit

when $$2\pi fL = \frac{1}{2\pi fC} \qquad (14F\text{–}1)$$

$$= \frac{1}{2\pi\sqrt{LC}} \qquad (14F\text{–}2)$$

$$C = \frac{1}{4\pi^2 f^2 L} \qquad (14F\text{–}3)$$

$$L = \frac{1}{4\pi^2 f^2 C} \qquad (14F\text{–}4)$$

Where L is in henries
C is in farads
f is in c. p. s

These are referred to as the necessary conditions for resonance and the current at resonance is given by the formula:

$$I_o = E/R \qquad (14F\text{–}5)$$

The current is in phase with the applied voltage. Theoretically with no resistance in the circuit the current is infinite.

Voltage Across Each Element of a Series Circuit

$$V_R = IR \qquad (15F\text{–}1)$$

$$V_L = 2\pi fLI \qquad (15F\text{–}2)$$

$$V_C = I/2\pi fC \qquad (15F\text{–}3)$$

Where I is computed from 13F-1

Coil and Condenser in Parallel Each with or without Resistance

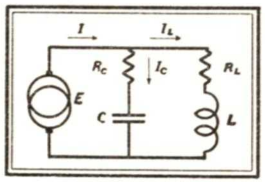

FIG. 6F

With R_c and R_L

$$I_c = E \Big/ \sqrt{R_c^2 + \left(\frac{1}{\omega C}\right)^2} \qquad (16F\text{–}1)$$

$$I_L = E/\sqrt{R_L^2 + \omega^2 L^2} \qquad (16F\text{–}2)$$

If R_c can be neglected:

$$I = \frac{E}{\left(\dfrac{X_c\sqrt{R_L^2 + X_L^2}}{\sqrt{R_L^2 + (X_L - X_c)^2}}\right)} \qquad (16F\text{–}3)$$

If R_L is small compared to X_L:

$$I = \frac{E}{\dfrac{L}{C} \times \dfrac{1}{\sqrt{R_L^2 + (X_L - X_c)^2}}} \qquad (16F\text{–}4)$$

When $X_L = X_c$, resonance:

$$I = \frac{E}{L/R_L C} \qquad (16F\text{–}5)$$

The factor $L/R_L C$ is the apparent impedance of a parallel resonant circuit, and

is purely resistive; i.e., the current I is in phase with the voltage E.

Practical Resonance Formulas

The general formula for the frequency at which resonance occurs in either series or parallel circuits is:

$$f = \frac{1}{2\pi\sqrt{LC}} \qquad (17F\text{–}1)$$

This formula is based on negligible circuit resistance, a condition safely assumed in practical radio circuits, and:

Where in 17F-1 f is in c. p. s.
L is in henries
C is in farads
R is in ohms

$$f = \frac{159.2}{\sqrt{LC}} \qquad (17F\text{–}2)$$

$$L = \frac{25{,}330}{f^2 C} \qquad (17F\text{–}3)$$

$$C = \frac{25{,}330}{f^2 L} \qquad (17F\text{–}4)$$

Where in 17F-2 to 17F-4
f is in kilocycles
L is in microhenries
C is in microfarads

Q Factor or Circuit Q

Quite often in discussing a series or parallel *resonant circuit,* the term Q is found. Since in practical series and parallel resonance circuits the circuit resistance is inherent in the coil, the merit of a coil is expressed by its Q factor.

$$Q = \omega L/R \qquad (18F\text{–}1)$$

$$Q = 1/\omega CR \qquad (18F\text{–}2)$$

Where L is in henries
R is in ohms
Q is a figure of merit
C is in farads

When R includes all circuit losses and the load, the Q factor is better termed Circuit Q. The Q factor represents essentially the voltage amplification in a coil by virtue of its series resonance or the impedance amplification in a coil by virtue of its parallel resonance.

G: COUPLED CIRCUITS

Basic Formulas

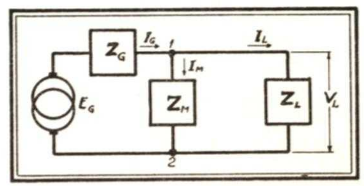

FIG. 1G

Given a general circuit with the coupling impedance Z_M, feeding a load whose impedance is Z_L. The generator has an impedance Z_G and generates a voltage E_G. In general coupled circuits and tube coupling, of primary importance, is the voltage across the load (V_L). For purposes of simple handling of the circuit, the equivalent impedance of Z_M and Z_L in parallel as viewed from the generator (terminals 1 and 2) is helpful. This is termed the primary equivalent impedance of the coupling device.

Impedance Reflected Into the Primary

$$Z_{12} = \frac{Z_M Z_L}{Z_M + Z_L} \qquad (1G\text{–}1)$$

Where Z_{12} is the impedance of Z_M and Z_L between terminals 1 and 2

Primary Current

$$I_G = \frac{E_G\,(Z_M + Z_L)}{Z_G Z_M + Z_G Z_L + Z_M Z_L} \qquad (1G\text{–}2)$$

Coupling Current

$$I_M = \frac{E_G\,Z_L}{Z_G Z_M + Z_G Z_L + Z_M Z_L} \qquad (1G\text{–}3)$$

Secondary or Load Current

$$I_L = \frac{E_G\,Z_M}{Z_G Z_M + Z_G Z_L + Z_M Z_L} \qquad (1G\text{–}4)$$

Load Voltage

$$V_L = \frac{E_G Z_M Z_L}{Z_G Z_M + Z_G Z_L + Z_M Z_L} \qquad (1G\text{–}5)$$

It is to be remembered that any impedance Z may be a device having a real and imaginary component ($Z = R + jX$). Or it may be a resistor or a reactance. Transformer coupling or network coupling may be reduced to an equivalent Z_M. Experts with the above basic formulas have derived formulas for practical circuits.* We shall consider only those that are regarded as most important.

Transformer Coupled

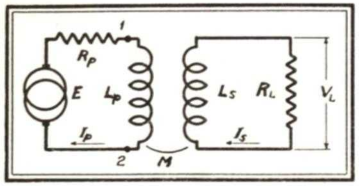

Fig. 2G

$$R_{12} = \frac{\omega^2 M^2}{R_L} \tag{2G–1}$$

$$I_P = \frac{E}{R_P + \dfrac{\omega^2 M^2}{R_L}} \tag{2G–2}$$

$$= \frac{E\,R_L}{R_P R_L + \omega^2 M^2}$$

Where, see Fig. 2G, L_P is small in comparison to L_S or unity coupling exists

Induced Secondary Voltage

$$E_S = \omega\, M\, I_P \tag{2G–3}$$

$$= E_S/R_L \tag{2G–4}$$

$$= \frac{\omega\, M\, E}{R_P R_L + \omega^2 M^2}$$

$$V_L = I_S R_L \tag{2G–5}$$

$$= \frac{\omega\, M\, R_L E}{R_P R_L + \omega^2 M^2}$$

When the coefficient of coupling K is known:

$$M = K\sqrt{L_P L_S} \tag{2G–6}$$

When K is equal to 1 (unity)

$$M = \sqrt{L_P L_S} \tag{2G–7}$$

Ideal Transformer Coupled (K = 1)

ωL_P and ωL_S are very large in comparison to R_P or R_L. See Fig. 3G. Transformers may be considered on the basis of turn ratio.

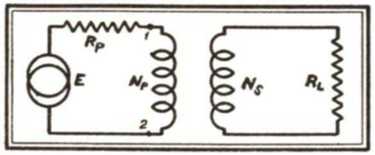

Fig. 3G

$$T_R = \frac{N_S}{N_P} = \frac{\text{Sec. Turns}}{\text{Pri. Turns}} \tag{3G–1}$$

Secondary Resistance R_L reflected into primary:

$$R_{12} = R_L/T_R^2 \tag{3G–2}$$

$$= R_L\left(\frac{N_P}{N_S}\right)^2 \tag{3G–3}$$

To match reflected R_L to R_P—condition for maximum power transfer when:

$$T_R = \sqrt{\frac{R_L}{R_P}} = \frac{N_S}{N_P} \tag{4G}$$

Reflected Impedance—Tuned Secondary

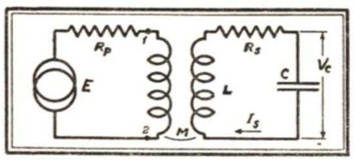

Fig. 4G

Case when C is tuned so I_S is a maximum.

Reflected Secondary Load into Primary is:

$$R_{12} = \frac{\omega^2 M^2}{R_S} \tag{5G–1}$$

Output Voltage

$$V_c = \frac{ME}{C} \times \frac{1}{R_P R_S + \omega^2 M^2} \tag{5G–2}$$

$$= \frac{\omega^2 MLE}{R_P R_S + \omega^2 M^2}$$

Maximum V_c when (optimum condition):

$$\omega M = \sqrt{R_P R_S} \tag{5G–3}$$

* For a more complete treatment of circuits containing L, C, R, and M refer to Henney's Radio Engineering Handbook, published by McGraw-Hill Book Co., New York City.

IT'S THE RUN THAT COUNTS

In baseball, the hero of the game is the man who scores. There are plenty of others who "almost" hit a home run—who "almost" scored—but these are forgotten men, as "almost" does not count.

First base—second base—third base—these are only stopping places on the road to a score. The world is full of stopping places, all guarded by other players equally bent upon winning. In the game of life, you must remember it is the run that counts—not the men "left on bases."

It's the fellow who knows *all* the rules—who is well trained and is prepared to take advantage of every opportunity who gets ahead. Don't be "left on base." Seize every opportunity to move forward—give the game everything that you have. Remember, no man can be stopped always—the fellow who keeps going is sure to win.

J. E. Smith